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NONLINEAR DYNAMIC RESPONSE CHARACTERISTICS OF A CLAMPED-CLAMPED BEAM SUBJECTED TO A UNIFORMALY DISTRIBUTED PERIODIC LOADING

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ABSTRACT

The nonlinear forced dynamic response of undamped thin beam with clamped-clamped configuration was investigated experimentally over a wide range of frequencies from 0-1400[Hz] and at different levels of excitation .An alloyed spring steel specimen with width 13mm, thickness 0.15mm and length 105mm was used and tests were conducted by pulsating the base frame sinusoidally with peak acceleration a[m/s^2] to apply uniformly distributed load. The response of the beam was obtained by measuring the relative displacement of the beam to the base frame.

An approximate equation of motion describing the nonlinear behavior of the beam was derived theoretically. The effect of damping as well as the anti-symmetric responses of the beam were neglected . The beam response , displacement and velocity, were monitored both in time and frequency domains. The nonlinear characteristics of the response such as sub and super harmonic, and internal resonances were investigated in relation to the level of excitation. Based on data collected samples of which presented graphically, conclusions were drawn concerning the beam dynamic response, jump phenomena and transition to chaos and their relation to the existing approximate theoretical models and experimental studies as well.

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CONTENTS

NOMENECLATURE

A Cross sectional area of the test beam [mm^2].

a Peak acceleration [m/s^2].

b Width of the test beam [mm].

E Young's modulus of elasticity [Gpa].

h Thickness of the test beam [mm].

I Moment of inertia of the cross section [m^4].

l Effective length of the test beam [mm].

P Load amplitude of the external force [N/m].

$\bar{P}$ Normalized load amplitude $=Pl^4/EIh$ [1].

P^* Nonlinear term due to the effect of transverse displacement of the external force caused by immovable supports [N].

t Time [s].

Y Transverse deflection of the test beam [mm].

y Normalized deflection of the test beam = Y/h [1].

ρ Mass density per unit length [$Kg/m^3/m$].

Ω Radian frequency of the external load [s^{-1}].

ω_n Linear circular frequency of the nth mode [s^{-1}].

ξ Non dimensional length = x/l [1].

δ Logarithmic decrement [1].

ϕ Orthonormal mode shape [1].

* Abbreviations :

RMS Root mean square.

* Symbols not listed are defined in the text.

CHAPTER 1

INTRODUCTION

Structural members subjected to vibration are often encountered in many engineering practices , such as rods underaxial load, shafts in torsion and beams in bending. The beams are frequently used in many engineering structures, therefore, the first step in understanding the dynamics of these structures is to study the dynamic response characteristics of beam elements. Of particular interest is the determination of the conditions under which a beam starts to behave in a nonlinear fashion, which occurs for example when the beam undergoes large amplitude resonance motion,. as such motions may lead to large internal deformations and usually catastrophic premature failure. As the amplitude of vibration becomes relatively large, the basic assumption of small amplitude, inherent in the linear theory is violated, and therefore, an analysis based on nonlinear theory becomes necessary for a more realistic prediction of the beam dynamic behavior.

1.1 FUNDAMENTALS

The study of a non-linear system is considerably more complicated than that of a linear one , and as a result the treatment of non-linear systems often requires entirely different methods of attack. It should be also recognized at the outset that the theory of non-linear vibrations is not nearly developed as well as that of the linear vibrations. In fact , it relies quite heavily on approximations based upon linear theory.

There are two basic approachs to non-linear systems , namely , qualitative and quantitative , Nayfeh and Mook (1979).The qualitative approach is concerned with the general stability characteristics of a system in the neighborhood of known solution , rather than with the explicit time history of the motion . On the other hand , the quantitative approach is concerned with just these histories. These solutions can be obtained by approximate methods such as perturbation methods or by numerical integration. Both qualitative and quantitative approachs are usually used to investigate nonlinear vibration as non of these approachs is sufficient to give a complete solution to these problems.

Nonlinearities may arise and enter the equations of motion by various ways, however, in general these nonlinearities may be classified into geometric , inertial and material , Nayfeh and Mook (1979) . Geometric nonlinearities may be caused by axial stretching of the median line in axially restrained beams or by large beam slopes in which the small angle assumption used in linear bending theory is no longer valid. This type of nonlinearity has been considered by Burgreen (1950), McDonald (1955) , Tseng and Dugundj (1970) , Eisley and Bennett (1970) , Takahashi (1976) , (1979) , Bennouna and White (1984) , Lewandowski (1987) , Liu , Kuo and Yang (1988), to mention a few. On the other hand nonlinear inertia effects may be caused by the presence of concentrated or distributed inertia elements, and by non-planar or parametric motions. Nonlinear beam elements of this type have been considered by, Haight and King (1972), Crespo da silva (1978) , Zavodney and Neyfeh (1989) and Hamdan (1991). Material nonlinearities occur whenever the stresses are nonlinear functions of the strains. The nonlinearities may also appear in the boundary conditions, Stupnicka (1983) , Moon and Shaw (1983).

It is generally known that exact analytical solutions to the nonlinear vibration of beams are difficult , so one has to rely on approximate solutions to these problems. A recent review of this subject is given by Hamdan (1991). The two commonly used methods are based on the assumption that the beam deflection is separable in space and time, Nayfeh and Mook (1979). In the first method the assumed solution is some form of truncated series in normal modes to which any of the variational methods such as Galerkin's method is applied , to reduce the original partial differential equations to the temporal modal equations of motion. If such series is limited to a single mode, then the method is known as the single mode approach which has a limitation that nonlinear dynamic coupling between various modes is neglected . Despite this limitation, this approach appears to lead to a resonable prediction of the response of such beam elements and it was used effectively by many authors , Zavodney and Nayfeh (1989) , Moon and shaw (1983) , Tseng and Dugundji (1970) Takahashi (1976) , Wagner (1965), and Hamdan (1990). On the other hand, the multiple modes approach has been used by some authors Yamaki and Mori (1980) , McDonald (1955) , Eisley and Bennett (1970), Takahashi (1979). It should be mentioned in the passing that the assumed single mode approach leads to a nonlinear deterministic Duffing's type oscillator (odd-powered type nonlinearity), while the assumed n modes lead to n nonlinear coupled Duffing's type oscillator.

The second common approach is based on the assumption that the time part solution is a harmonic function , and then one applies the harmonic balance method to obtain the spatial nonlinear boundary value problem, Bennouna and white (1984) , Lewandoski (1987) , Nagesararao (1988). Iterative techniques , finite element and finite difference methods have been widely used to solve relevant nonlinear boundary value problems, Takahashi (1976) ,Mei (1973) , Rao and Raju (1976).

Throughout the vast amount of researchs conducted on the nonlinear beam dynamic response, many of the nonlinear dynamic characteristics are known such as jump phenomena, sub and super harmonic resonances. However,to a variety of nonlinear phenomena arising in such physical systems which are usually governed by simple deterministic Duffing's equation. A distinctly new class of motion called chaotic motion , has been added recently. These consist of bounded and unstable periodic motions with random-like behavior which may persist for a long period of time. These motions occur only undercertain conditions of forcing amplitude and frequency which are distinguished from the known stable harmonic and periodic motions in the two dimensional phase-plane, these motions are characrarized by many closed trajectories, whereas a single periodic solution is characterized by a single closed orbit.Chaotic behavior in nonlinear oscillators was first reported by Ueda (1979),(1980),(1981).Deterministic systems exhibit continuous, bounded motions (chaos) that can be found in many engineering systems such as the vibration of buckled beams and plates , Moon (1980) , Tongue (1986) , Pazehki and Dwell (1987) . These motions occur when the vibration amplitude becomes large enough to cause the beam to " snap-through ". Of a particular interest is the determination of the conditions under which such chaotic motions may occur.

1.2 PRESENT WORK OBJECTIVES

Although a significant progress has been made to predict these chaotic motions theoretically, Stupnika and Bajkowski (1986), Stupnika (1987), Ueda (1981), i.e in connection with sub and super harmonic instability analysis, most of the existing analysis of chaotic behavior is based on qualitative methods.For example these motions are studied by monitering the response wave form, frequency spectrum and phase-plane plots, therefore, one may determine the conditions under which these motions occur.

The object of this work is to attempt experimentally to shed more light on some nonlinear dynamic behavior of a typical clamped-clamped thin beam. Nonlinear response characteristics of the beam such as sub and super harmonics, jumps and chaos will be studied. Emphasis is placed on the effect of level of excitation on these phenomena , since there seem to be a lack of experimental data on this subject. Correlation of experimental data with existing theoretical models will be attempted, whenever possible in order to clarify some of the physical aspects of the problem.

1.3 ORGANIZATION OF THE THESIS

The thesis is divided into six chapters , of which this introduction the first. Chapter (2) is a review of the previous related literature . In chapter (3) a basic review of the known theoretical dynamic behavior of nonlinear beam is presented. In chapter (4) the experimental set-up and the measuring techniques,as well as , the experimental procedures are discussed in detail. The results of experimental work are presented graphically and discussed in chapter (5). Finally the information gained from experimental work on the effects of level of excitation on dynamic response and transition to chaos are drawn together in chapter (6) , which contains the remarks and recommendations for further work on this problem .

CHAPTER 2

REVIEW OF PREVIOUS WORK ON NONLINEAR BEAM VIBRATION AND CHOATIC MOTION .

2.1 INTRODUCTION

In the theory of small linear vibrations of a beam, it is found that the complete motion can be described as the sum of infinite number of separate , distinct sinusoidal modes. The modes are statically and dynamically independent of each other if there is no damping or if the damping coefficients are in proportion to the mass and stiffness. The amplitude and frequency of a given mode are neither functionally related nor are the amplitudes and the frequencies of various modes dependent upon each other, McDonald (1955), Burgreen (1950).

These conditions do not hold in the vibration of a beam whose equations of motion are essentially nonlinear. A typical example to investigate such equations is provided in the case of uniform beam whose ends are axially restrained, McDonald (1955),Takahashi (1979), Liu, Kuo and Yang (1988).The nonlinear effect in the beam is produced by the axial stretching of the beam nuetral axis. A discussion of the problem of the vibration of uniform beam with axially restrained edges provides a technical information about the amplitude and frequencies of the vibration of such a beam. This information may be of value to machine and structural designers since the analysis shows that significant changes in the frequencies occur if the beam ends are restrained .

The aim of this chapter is to review some of the investigations that have been reported on the nonlinear beam vibrations with clamped ends and its related factors. In particular, the cases where nonlinearity arises from mid-plane stretching, and to report some of the studies conducted on choatic oscillation .

2.2 NONLINEAR BEAM VIBRATION

McDonald (1955) investigated the vibration of a uniform beam with axially restrained hinged edges. The beam is subjected to a concentrated force at its mid-span, and with arbitrary initial conditions, exhibited a dynamic coupling of its modes of vibrations. The frequencies of various modes are functionally related to the initial conditions , particularly the amplitudes of all modes . Eringen (1952) , and Chu and Herrmann (1956) derived an approximate solution to the equation of motion for large vibration amplitudes for simply supported beams and plates and predicted the appearance of cubic stiffness term in the differential equations of motion in each mode . Smith , Malme and Gogos (1961) investigated the nonlinear behavior of clamped-clamped thin Aluminum strip under sinusoidal pressure excitation. Their experimental measurements showed that the ratio of the strain at the end of the strip to that at the center was dependent on the amplitude of vibration,which implies that the fundamental mode shape is much flatter at mid-span and has higher curvature at the ends at large vibration amplitudes. It was suggested that the steady state forced response could be more accurately predicted if the flexural displacement could be expressed as the sum of at least two modes , the relative proportions of which would change with the level of excitation.

Tseng and Dugundji (1970) experimentally and analytically investigated the nonlinear vibrations of a clamped-clamped beam excited by sinusoidal motion of its supports in a direction normal to its span . Applying the harmonic balance method to solve Duffing's equation , they found that super-harmonic response of orders 3/2 ,2,3,.... as well as sub-harmonic responses of orders 2/3,1/2,1/3 could occur in addition to the fundamental response.Multiple solutions may exist depending on the initial conditions. Their results indicated that , for large amplitudes of vibration , the general solution involved the forcing frequency component as well as the super-harmonic and sub-harmonic components.

Busby and Weingarten (1972) examined the effects of multi modal participation for simply supported and clamped-clamped beams , using the finite element technique to formulate the nonlinear differential equation. The averaging method was used to obtain an approximate solution which was compared with results of solutions obtained by using an analog computer. It was found that the phenomenon of coupled responses will occur if the nonlinear stiffness terms are large. Takahashi (1976) studied the nonlinear free vibration of a clamped-clamped beam with immovable ends. His work considers the effect of multiple mode participation and higher harmonics of motion. This case was analyzed by application of Galerkin's method. The deflection of the beam is an assumed series of the product of the normal modes of a clamped-clamped beam and unspecified function of the time. The resulting nonlinear ordinary differential equation of motion was solved by the harmonic balance method. Multiple mode approach , single mode approach and finite difference methods solutions were presented and those solutions were compared for various amplitude ratio of a clamped-clamped beam . The main conclusions of his work were that the increase in the forcing amplitude considerably affected the nonlinear vibrations of a clamped-clamped beam.Also the third harmonic was found to play a significant role in the first natural frequency range. In particular when the beam is subjected to a periodically varying load is on the beam due to parametric excitation through nonlinear coupling terms.

The nonlinear vibrations of a clamped-clamped beam with initial deflection and initial axial displacements has been studied theoretically and experimentally by Yamaki, Otomo and Mori (1980). Their study clarifies the general features of the nonlinear response of beams under harmonic excitation.The solutions have been obtained theoretically by the authors in an approximate manner,by assuming the beam to be a three degrees of freedom system and by neglecting the effect of damping as well as that of anti symmetric responses of the beam. The main results can be summarized as follows :

a) Theoretical predictions are generally in reasonable agreement with experimental ones , but there are some discrepancies between theory and experiment , significant with the increase in amplitude.

b) Nonlinear responses become much more complex in cases when the beam is buckled under the initial axial displacement.

c) In addition to the responses theoretically predicted,a variety of nonlinear responses have been found to occur in connection with the internal and combination resonance,as well as dynamic snap-through phenomena.

The effect of large vibration amplitude on the fundamental mode shape of a clamped-clamped uniform beam was studied by Bennouna and White (1984). The results were expressed with the fundamental resonance frequency as a function of the amplitude to beam thickness ratio . They found that the theoretical values were higher than the experimental ones due to the flexibility of the lamps and the coil mass.

Although solutions for the steady state problem have been found , there is assurance neither that these are unique nor that they are stable solutions because of the nonlinearity of the equations. Consequently , it is necessary to check the stability of these solutions and to investigate the possibility of further steady state solutions , Bennett and Eisley (1970). The stability of the solutions is investigated by studying the behavior of a small perturbation of the steady state response. The perturbation equations in general result in coupled Hill-type equations. The stability problem has received very little attention in the past , although Bennett and Eisley (1970) analyzed the coupled Hill type equations by a direct application of Floquet theory. Busby (1971) investigated them by means of the averaging method. However , for the coupled Hill-type equations it seems that there exist no convenient methods of analysis .

Takahashi (1980) made a stability analysis of the steady state response for the large amplitude of a nonlinear beam with a clamped-clamped configuration under periodic excitation using the multiple degree-of-freedom approach. The unstable amplitudes of the first and third modes were plotted in the stability diagram theoretically and unstable regions were located experimentally, especially the unstable boundaries of the third super-harmonic response of the first mode.

All of the above studies use Duffing's equation model which may be written in the following form see, i.e,Meirovitch (1986) :

$$\ddot{x} + 2\xi\dot{x} + x + \epsilon x^3 = P_o \cos \Omega t \ , \quad \omega_n = 1 \qquad (2\text{-}1)$$

where :

ξ = damping coefficient.
ϵ = small given parameter.
P_o= amplitude of the harmonic external force.
Ω = driving circular frequency.

This type of Duffing's equation appears in various physical and engineering systems (e.g nonlinear beam vibration with geometric nonlinearity), and this is one of the simplest and most important nonlinear differential equations. There are various types of steady motions exhibited by Duffing's equation . Among them deterministic or regular motions are generally known , e.g , harmonic , super harmonic and sub harmonic motions . However , owing to the perfectly deterministic nature of the equation , no reference has been made to the possibility of the existence of chaotic motions for a long time . The occurrence of chaotic motions was originally studied by Ueda (1973),(1978),(1979). The author made a survey of the steady motions exhibited by Duffing's equation using analog and digital computer simulation. The obtained solutions were examined and regions of regular and chaotic motions were determined at different initial conditions and system parameters such as damping and load amplitude of the external force. Regions at which sub and super harmonics sustained in relation to the occurance of chaotic motions were located depending on the phase-plane plots. Samples of chaotic and periodic motions are shown in Fig.(2-1).

Moon (1980) conducted a study in which the existance of strange attractors was experimentally demonstrated, e.g , generalization of a limit cycle .

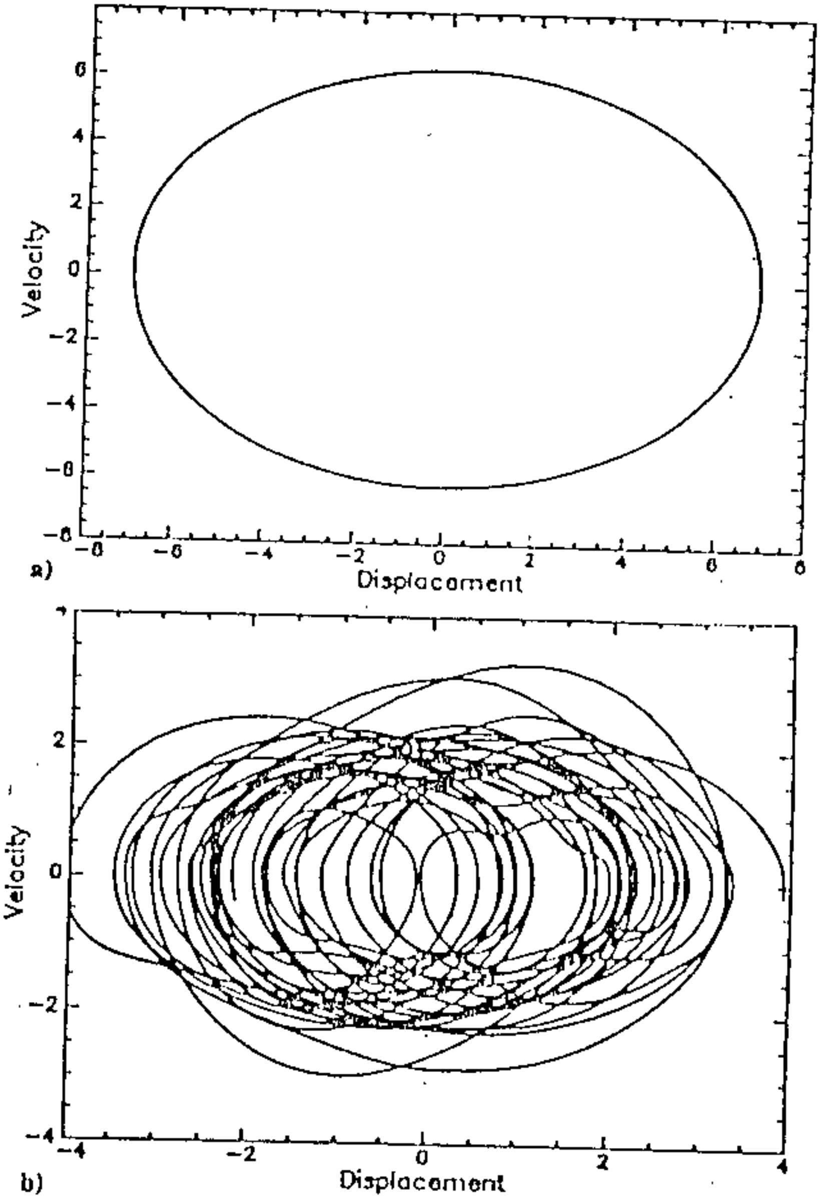


Fig.(2-1): Numerically calculated phase-plane trajectories a) periodic b) chaotic motion, Moon (1990).

The experimental apparatus consists of a cantilevered beam that was buckled with magnetic forces. In this case the motion consisted of the beam snapping from one buckled position to another in chaotic manner. Moon also determined an experimental threshold criteria for chaotic motion by studying the effect of the forcing amplitude at certain forcing frequencies. The main conclusion was that at sufficiently high amplitude of the external forcing, chaotic motion would occur and such motion might not persist. However, a threshold might occur when the beam jumps spontaneously out of periodic to chaotic motion, this threshold is shown in Fig.(2-2).

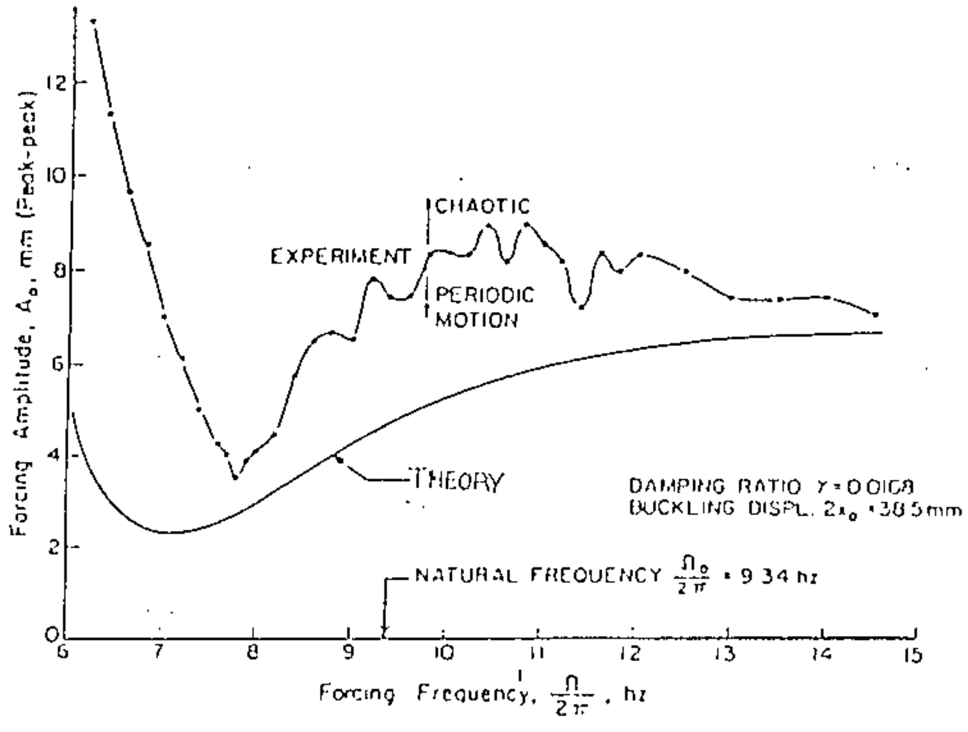


Fig.(2-2): Experimental and theoritical thresholds for chaotic motion for a beam buckled by magnetic forces, Moon (1980).

Stupnicka and Bajkowski (1986) have studied the 1/2 sub harmonic resonance and its transition to chaotic motion in a nonlinear oscillator with unsymmetric nonlinear force. Chaotic motion has also been observed for the classical `Duffing equation with positive cubic nonlinearities , i.e , hardening nonlinearity . Stupnika (1987) has studied theoretically two types of nonlinear oscillator i.e symmetric and unsymmetric elastic characteristics in an attempt to trace routes to choas for both types of the forced oscillators having single equilibrium positions . The main observations of her work were that the zone of chaotic motion occurs in the neighborhood of the theoretical stability limit of the 1/2 sub harmonic resonance: i.e, close to the frequency where resonance curves have vertical tangent. Also another observation was that the narrow chaotic zones occur near the jumps could be preceded by a cascade of period doubling as obtained from simulation results.

More recently, Moon and Li (1990) studied experimentally the chaotic vibratons in a pin-jointed space truss structure. Their results demonstrated that under periodic excitations, the truss response exhibited a chaotic-like behavior, see Fig.(2-1). The dynamics of the truss becomes more complicated by nonlinear pin-joints. The degree of chaos was lowered by adding a tension cable along the longeron direction of the truss. Criteria for chaotic motion was obtained in the truss as shown in Fig.(2-3), in which the horizantal axis represents the driving frequency while the vertical axis is the acceleration of the shaker. The graph also shows the effect of cable tension on the occurence of chaotic motion.

It might be concluded from the afore mentioned studies that transition from regular to chaotic motions in a deterministic Duffing's oscillator occur at a certain parameters of the equation of motion. Of particular importance are the amplitude of the external load applied and the level of damping. Also this transition from regular motions to chaotic ones could be preceded by a series of period doubling related to higher harmonics. In the present experimental work, the effect of varying the level of excitation over a certain frequency ranges will be studied to attempt to predict such transition from regular to chaotic motions using the two dimensional phase-plane, frequency spectrum and time histories of motion.

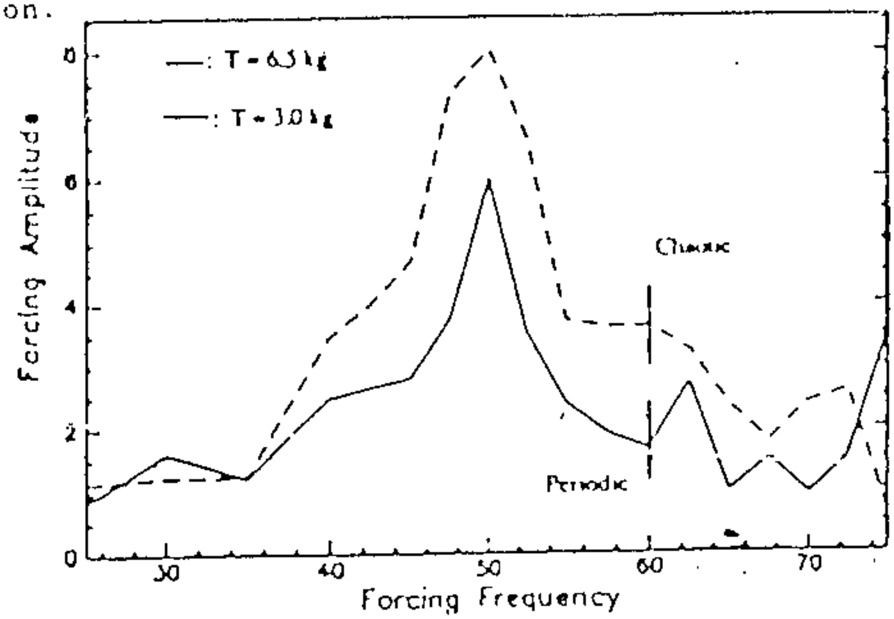


Fig.(2-3): Criteria for chaos in a space truss, Moon & Li (1990).

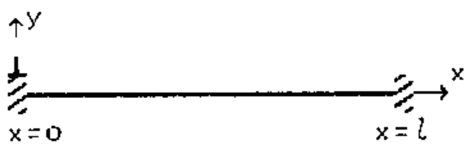
CHAPTER 3

REVIEW ON THE THEORETICAL NONLINEAR DYNAMIC BEHAVIOR OF CLAMPED-CLAMPED BEAMS

3.1 SURVEY

One of the most fundamental problems in the nonlinear vibrations of structures, is the nonlinear response of a beam under periodic lateral loading , whose edges are restrained from axial displacement. As mentioned previously the solution to this problem leads to nonlinear Duffing type oscillator. In the following theoretical review , following similar analysis made by Yamaki and Mori (1980) , the effect of damping as well as anti symmetric modes are neglected in the theoretical derivation of the equation of motion

3.2 BASIC EQUATIONS AND BOUNDARY CONDITIONS



Assume that a uniform beam with cross sectional area A , moment of inertia I and length l is subjected to the uniformly distributed periodic lateral load $P\cos\Omega t$.

The governing equations of motion for the transverse vibration of this straight beam which is axially restrained and large deflections are permitted can be expressed as follows , according to Takahashi (1979) :

$$P^{*} = -\frac{EA}{2l}\int_{0}^{l}\left(\frac{\partial y}{\partial x}\right)^{2}dx \qquad (3\text{-}1)$$

$$L(Y,P) = EI\frac{\partial^{4}Y}{\partial x^{4}} + \rho A\frac{\partial^{2}Y}{\partial t^{2}} + P^{*}\frac{\partial^{2}Y}{\partial x^{2}} = P\cos\Omega t \qquad (3\text{-}2)$$

For this case of clamped-clamped configuration the boundary conditions become :

At $x=0$, $Y=\frac{dY}{dx}=0$

At $x=l$, $Y=\frac{dY}{dx}=0$

If $P^{*}=P=0$, then equation (3-2) will be reduced to the linear eigen-value problem describing the beam motion which can be solved by the classical methods such as the separation of variables technique. After the application of the clamped boundary conditions to the obtained solution, the following frequency equation for the beam is obtained :

$$1 - \cos \beta_n l \cosh \beta_n l = 0 \qquad (3-3)$$

where:

$$\beta_n l = \omega_n^{1/2} \left(EI/\rho l^4 \right)^{-1/4} \qquad (3-4)$$

In order to transform equation (3-2) into a dimensionless form, the following notations are introduced assuming that the beam has a rectangular cross-sectional area with width b and thickness h :

$$\xi = x/l \quad , \quad \bar{P} = \frac{P l^4}{EIh} = \frac{12\ l^4 P}{Ebh^4} \quad , y = Y/h$$

$$\Omega_o = (1/l^2)(EI/\rho h)^{1/2} \quad , \quad \omega = \Omega/\Omega_o \quad , \quad \tau = \Omega_o t$$

With the above notation , equation (3-2) could be rewritten as:

$$L(y) = \frac{\partial^4 y}{\partial \xi^4} + \frac{\partial^2 y}{\partial \tau^2} - 6 \int_0^l \left(\frac{\partial y}{\partial \xi}\right)^2 d\xi \, \frac{\partial^2 y}{\partial \xi^2} - \bar{P} \cos\omega\tau = 0 \qquad (3-3)$$

$$\text{At} \quad \xi = 0\ ,\ 1 \ : \quad y = 0 \quad , \quad \frac{\partial y}{\partial \xi} = 0 \qquad (3-4)$$

3.3 GENERAL METHOD OF SOLUTION

The method used in the solution is the assumed mode shape method. The initial value problem is obtained by assuming a series solution satisfies the clamped boundary conditions :

$$y = \sum q_m(\tau) \phi_m(\xi) \qquad m= 1,2,3,.. \qquad (3-5)$$

where :

y = Beam deflection.

$q_m(\tau)$ = Unknown time function.

$\phi_m(\xi)$ = Orthonormal natural modes of the vibration whose both edges clamped : that is

$$\left.\begin{aligned} &\phi_m(\xi) = \cosh\alpha m\xi - \cos\alpha m\xi - \frac{\sinh\alpha m + \sin\alpha m}{\cosh\alpha m - \cos\alpha m}(\sinh\alpha m\xi - \sin\alpha m\xi) \\ &\& \\ &1 - \cosh\alpha m \cos\alpha m = 0 \end{aligned}\right\} \quad (3-6)$$

Further :

$$\left.\begin{aligned} &\frac{\partial^4 \phi m}{\partial \xi^2} - \omega_m^2 \phi_m = 0 \quad , \quad \omega_m = \alpha_m^2 \\ &\int_0^1 \phi_m \phi_n \, d\xi = \delta_{mn} = \begin{cases} 1 \text{ if } m=n \\ 0 \text{ if } m \neq n \end{cases} \quad m,n=1,2,3... \end{aligned}\right\} \quad (3-7)$$

The first five of the functions $\phi_m(\xi)$ are depicted in Fig.(3-1) . Note that $\phi_m(\xi)$ with m=1,3,5,.. are symmetric while those with m=2,4,6,.. are anti-symmetric with respect to the mid-point $\xi=1/2$.

Applying Galerkin method to equation (3-3) which leads to:

$$\int_0^1 L(y)\ \phi n(\xi)\ d\xi = 0 \quad , \quad n=1,2,3 \qquad (3\text{-}8)$$

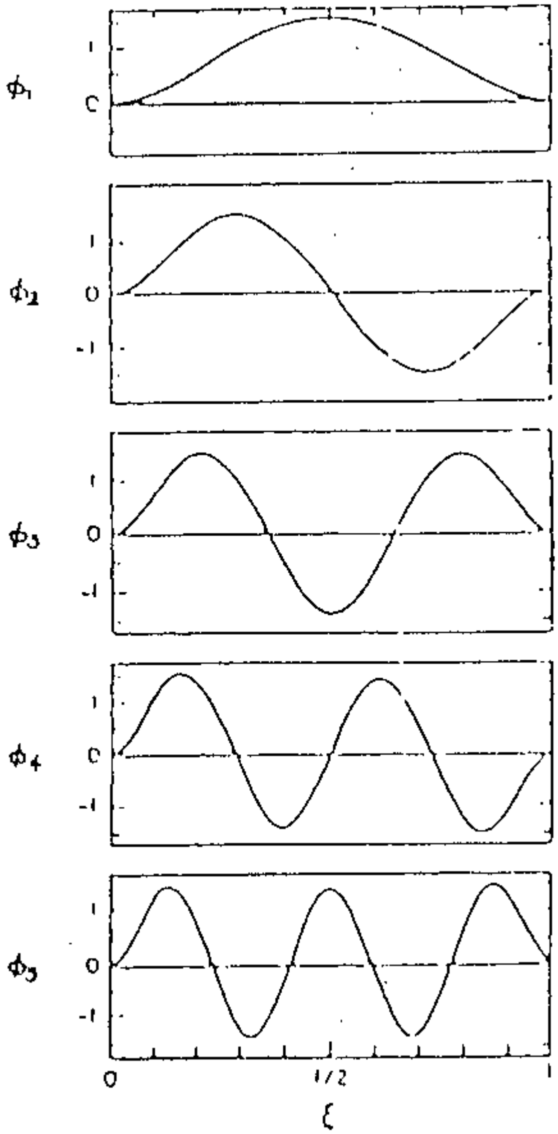


Fig.(3-1): Normal modes of vibration, ϕ_n(n=1-5) for the clamped-clamped beam.

Performing the integration gives :

$$\omega_n^2 q_n + \frac{\partial^2 q_n}{\partial \tau^2} + 6 \sum_k \sum_l \sum_m \beta_{kl} \beta_{mn} q_k q_l q_m = \gamma \bar{P} \cos\omega\tau \qquad (3-9)$$

where :

$$\beta_{mn} = \int_0^1 \frac{\partial^2 \phi_m}{\partial \xi^2} \phi_n d\xi = - \int_0^1 \frac{\partial \phi_m}{\partial \xi} \frac{\partial \phi_n}{\partial \xi} d\xi = \beta_{mn}$$

$$\gamma_n = \int_0^1 \phi_n d\xi \qquad (3-10)$$

It is noted from symmetry , $\beta_{mn} = 0$ when m+n is odd while $\gamma_n = 0$ when n is even.

Equations (3-10) represent a set of coupled Duffing equations in terms of $q_m(\tau)$; Yamaki and Mori (1980).

Equation (3-9) could be rewritten assuming two symmetrical mode vibration as :

$$\left.\begin{aligned}&\frac{\partial^2 q_n}{\partial \tau^2} + \omega_n^2 q_n + 6\,\Delta_n = \gamma_n \bar{P} \cos\omega\tau \qquad n=1,3\\&\text{where :}\\&\Delta_n = \sum_{k=1,3} \sum_{l=1,3} \sum_{m=1,3} \beta_{kl}\,\beta_{mn}\, q_k q_l q_m\end{aligned}\right\} \quad (3\text{-}11)$$

$$\left.\begin{aligned}\Delta_1 &= \beta_{11}^2 q_1^3 + 3\,\beta_{11}\beta_{31}\, q_1^2 q_3 + 2\beta_{13}^2\, q_1 q_3^2 + \beta_{11}\beta_{33}\, q_1 q_3^2 + \beta_{31}\beta_{33}\, q_3^3\\\Delta_2 &= \beta_{13}\beta_{11} q_1^3 + 3\,\beta_{13}\beta_{33} q_1 q_3^2 + 2\,\beta_{13}^2 q_1^2 q_3 + \beta_{11}\beta_{33} q_1^2 q_3 + \beta_{33}^2 q_3^3\end{aligned}\right\} \quad (3\text{-}12)$$

Relevant values of the coefficients in equation (3-11) are as follows , Yamaki and Mori (1980)

$$\begin{aligned}&\omega_1 = 22.37 , && \omega_3 = 120.9\\&\gamma_1 = 0.8307 , && \gamma_3 = 0.3638 && (3\text{-}13)\\&\beta_{11} = -12.3 , && \beta_{13} = 9.731\\&\beta_{33} = -98.9\end{aligned}$$

Substituting numerical values listed above , result in two coupled Duffing equations of the form :

$$\frac{\partial^2 q_1}{\partial \tau^2} + 500.417\ q_1 + 907.8\ q_1^3 - 2154.6\ q_1^2\ q_3 + 8436 q_1 q_3^2 +$$

$$+ 5774.4 q_3^2 = 0.83 \bar{P} \cos\omega\tau \qquad (3\text{-}14)$$

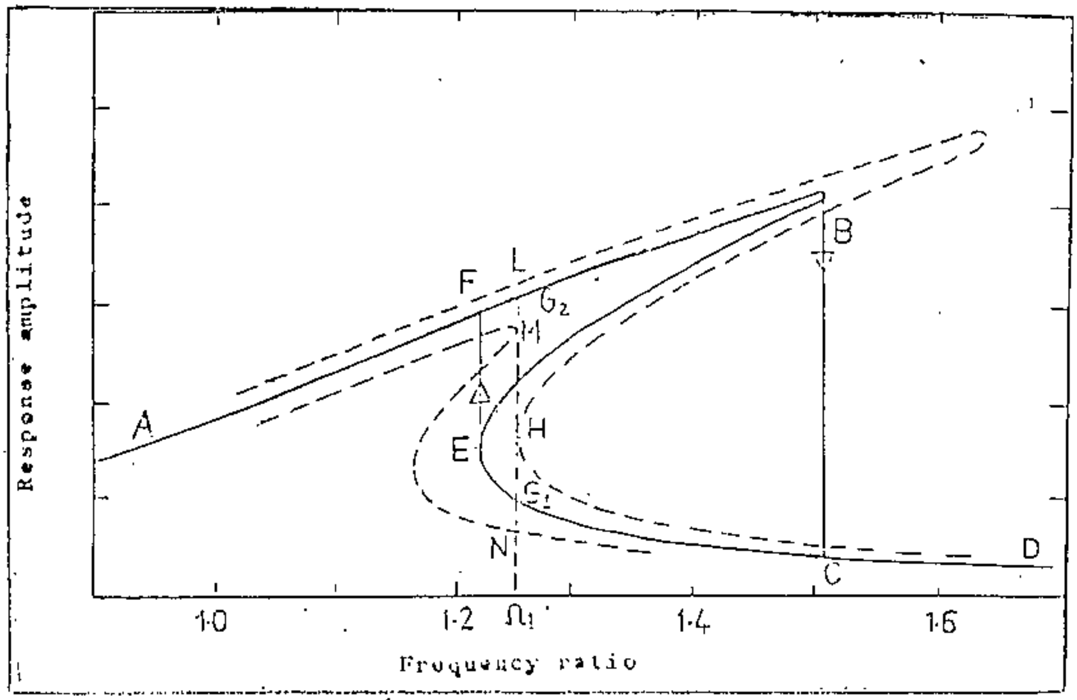
$$\frac{\partial^2 q_3}{\partial \tau^2} + 14616.8\ q_3 - 718.2 q_1^3 + 8436\ q_1^2 q_3 - 17323 q_1 q_3^2$$

$$+ 58687.2 q_3^3 = 0.363 \bar{P} \cos\omega\tau \qquad (3\text{-}15)$$

These coupled Duffing equations were solved by Yamaki and Mori by assuming a steady state solution of the form :

$$q_m = \sum_{j=0}^{3} a_m^{j\mu} \cos(j\mu\omega\tau) \quad , m=1,3 \quad , \mu = 1, 1/2, 1/3, .. \quad (3\text{-}16)$$

and applying the balance harmonic method. For given values of $\bar{P}$, ω ,and μ, they obtained a set of cubic simultaneous equations for the determination of $a_m^{j\mu}$ which solves the problem. This theoretical model will be used to explain the experimental resultsin the following chapters. Since this theoretical model results in n-coupled Duffing's equations, detailed and accurate solutions are beyond the scope of this thesis as this involves complex mathematical manipulations.Solutions, however are known for the simple case of a single degree of freedom Duffing's oscillator (n=1). This solution is shown in Fig.(3-2) which shows a typical form of response amplitude as a function of excitation frequency in the neighborhood of the linear resonant frequency.



Fig(3-2): Nonlinear forced response of a system modelled by Duffing equation, Robertson (1984).

Response curves are shown for three levels of exciting force. Well-known aspects of the response curves are the amplitude jump phenomena-rapid rise or collapse of response at points in the resonance region which is shown in the path ABCD that is followed for increasing frequency with a collapse at BC, while reversing of forcing frequency the path DEFA would be followed with an incremental jump at EF. It is also seen that two stable response amplitudes may be obtained for a given force and the system may be switched between these, i.e, points G_1 and G_2, by means of an impulse.

The Duffing oscillator given by equation (3-15) with n=1, can also exhibit similar resonance responses as those in Fig.(3-2) at forcing frequencies which are multiple (super-harmonic) or fractions (sub-harmonic) of the forcing frequency.

CHAPTER 4

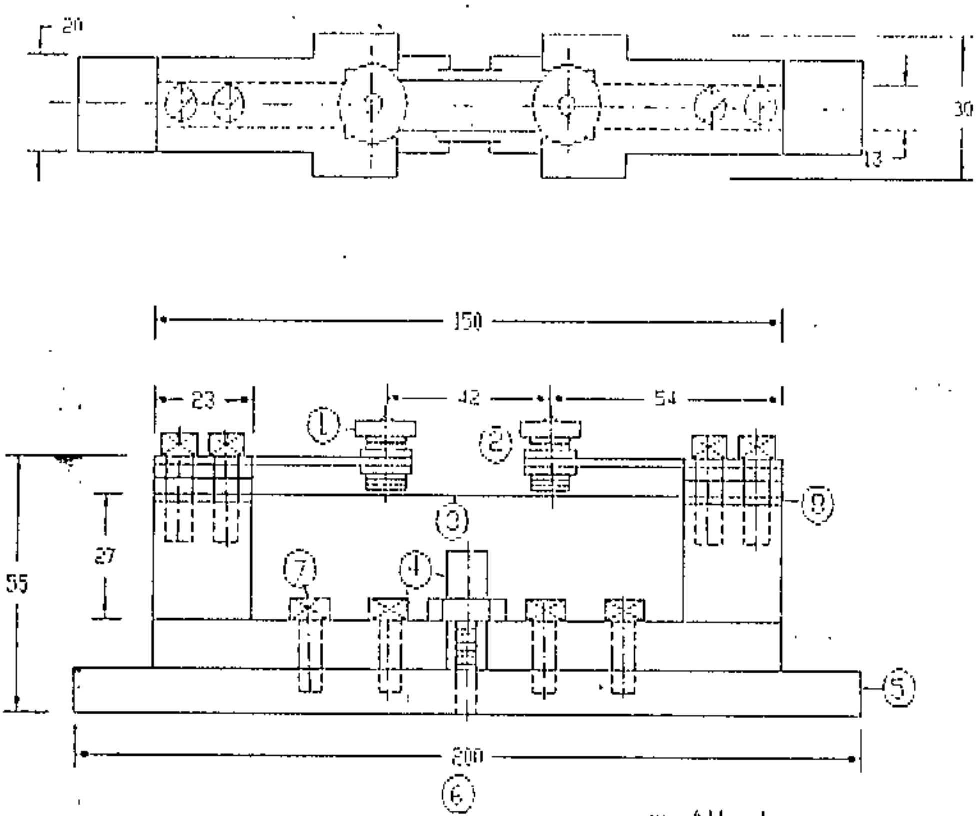
APPARATUS AND INSTRUMENTATION

4.1 INTRODUCTION

The experimental investigation for the nonlinear dynamic response of a clamped-clamped beam subjected to a uniformly distributed periodic loading over a wide range of frequencies at different levels of excitation was conducted. In the following sections, the experimental test setup as well as the different instrumentations and measurement techniques, used to carry out the experimental investigations, are described in detail.

4.2 TEST SPECIMEN AND BASE FRAME

A test beam with 105mm in length, 13mm in width and 0.15mm in thickness made from spring alloyed steel is used . The Young's modulus E and the mass density ρ were found to be $E = 168*10^9\ N/m^2$ and $\rho = 7.67*10^3\ Kg/m^3$. The initial deflection of the beam was found to be almost negligible. In addition, the beam material was analyzed using atomic wave absorption test. The chemical composition of the alloy are (0.01% Mo, 2.59% Ni, 0.06% Cr, 0.34% NB, 1.72% Cu). The base frame was fabricated from a commercial mild steel in the workshop of the faculty of engineering and technology, University of Jordan.



* All dimensions are in mm.

Part list :

No.	Quantity	Name	Material
1	1	Vel pick-up	Stain steel
2	1	Displ. pick-up	Stain steel
3	1	Test beam	Spring steel
4	1	Acceleromrter	Stain. steel
5	1	Base frame	Mild steel
6	1	Exciter head	Steel
7	8	M6 bolts	Steel
8	2	Stiff. sheet	Aluminum

Fig.(4-1): Schematic diagram of the test beam,base frame and transducers.

As can be seen in Fig.(4-1), the clamped portions of the beam were stiffened with a pair of 1mm aluminum sheets which arefirmly fixed to the mild steel base frame through two 6mm thick steel plate by two M6 steel bolts. The base frame is bolted at the middle to the exciter head. By shaking the base frame sinusoidally with radian frequency Ω and peek acceleration a[m/s^2], a uniformly pulsating distributed load, $P\cos\Omega t$ can be applied to the beam, the load amplitude being $P=\rho bha = 14.95*10^{-3}a(N/m)$

4.3 TEST EQUIPMENT AND INSTRUMENTATION

Tests were conducted by exciting the base frame of the beam periodically and measuring the relative displacement and velocity of the beam to the frame by using two contactless dynamic pickups (B&K MM0004 and B&K MM0002) respectively. The mounting of the test beam to the vibration exciter (B&K 4808) and the arrangement of the transducers are shown in Fig.(4-2). It can be seen that both transducers facing the beam are located at equal distances from the midspan of the beam. A piezoelectric accelerometer (B&K 4371) is used to monitor the acceleration of the exciter head. A schematic diagram of the whole test set up is shown in Fig.(4-3). The output signal from the pickups was simultaneously fed to a digital frequency analyzer (B&K 2131) and to measuring amplifiers (B&K 2616), then two tunable band pass filter (B&K 1621) and to a digital storage oscilloscope (OS 4100). Using this arrangement, it was possible to measure the R.M.S value of the amplitude of oscillations and to monitor the signals on both the frequency and time domains. The signals from the storage oscilloscope was recorded using an X-Y recorder type (wx 4402).

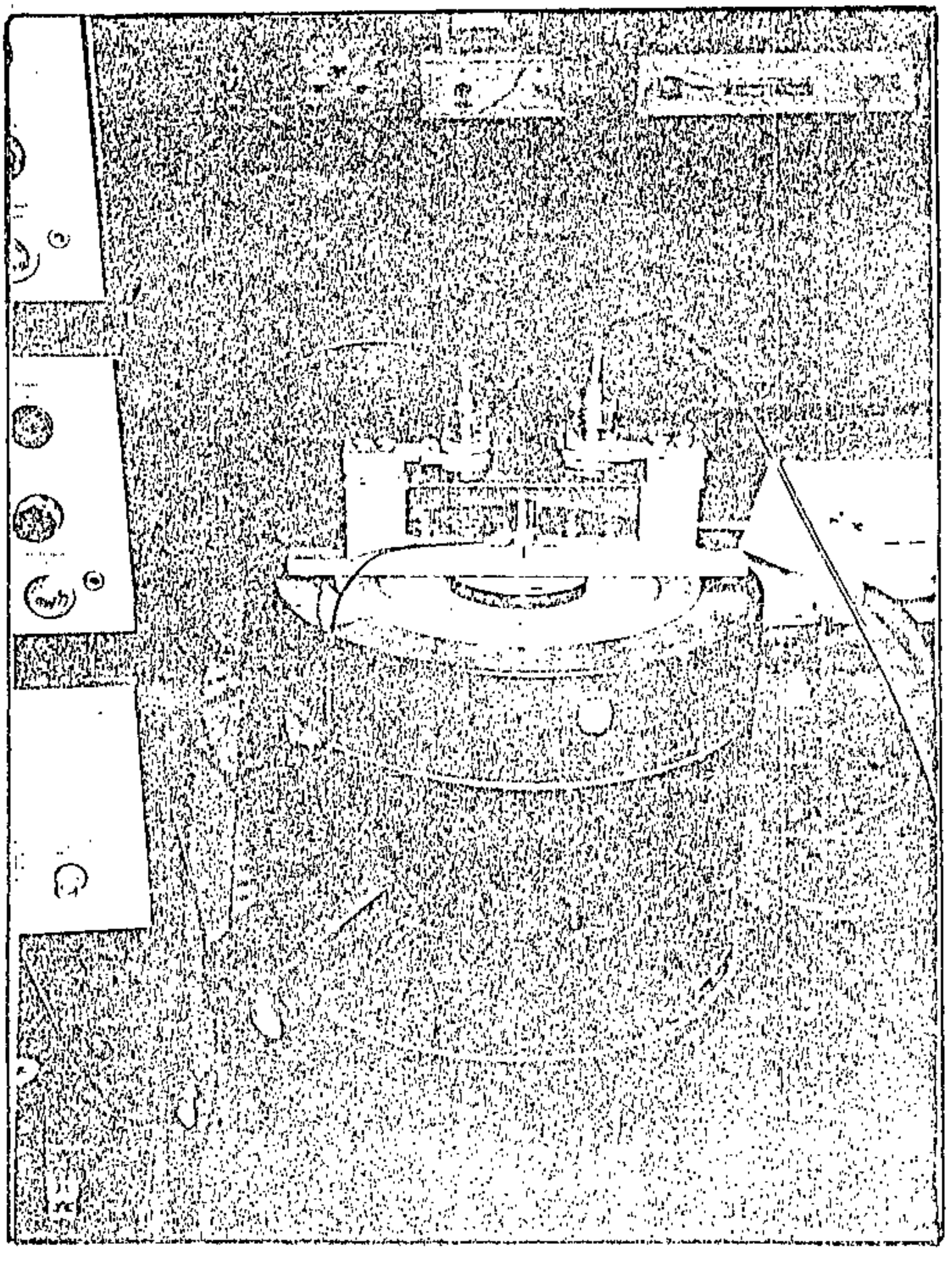


Fig.(4-2): A view of the test model.

The dynamic characteristics and calibration of the dynamic pickups and the processing equipment can be seen in the manuals provided by manufacturers . Since many different equipment were used in the present work it is not possible to report their characteristics and their calibration procedures. A general view of the experimental set-up is shown in Fig.(4-4).

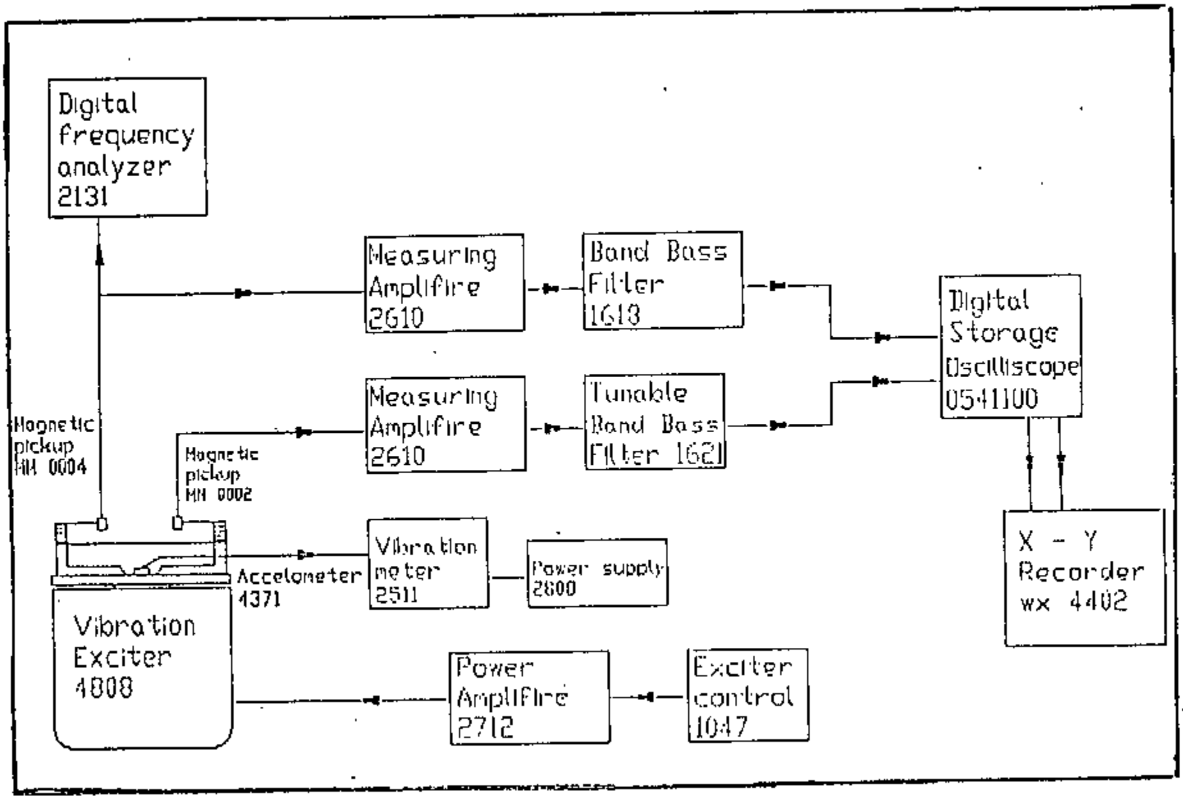


Fig.(4-3): Schematic diagram of the experimental set-up.

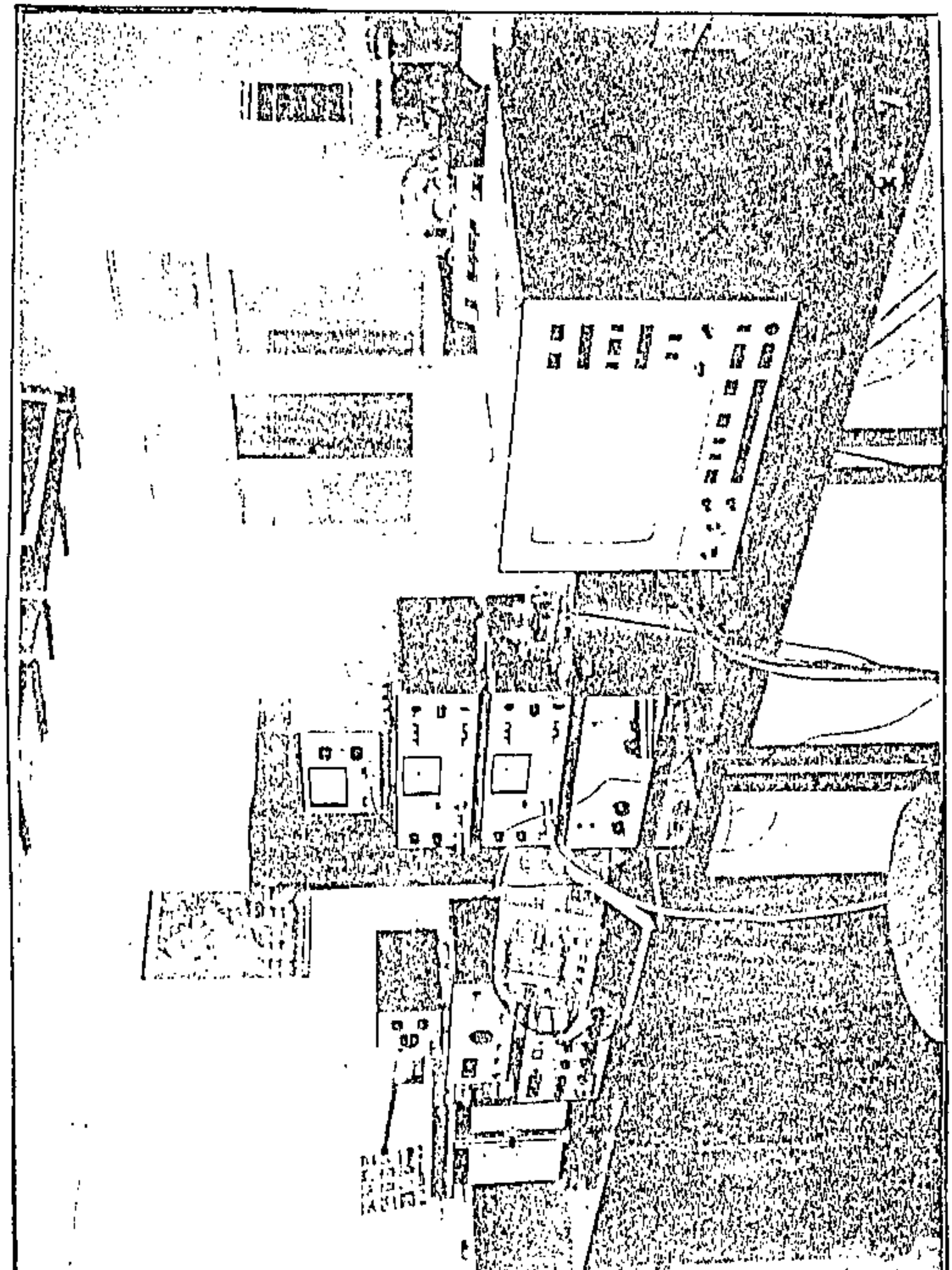


Fig.(4-4): General view of the experimental set-up.

4.4 TEST PROCEDURES

The experiments were conducted using the magnetic pickup (MM 0004) located at $x/l \simeq 0.3$ for measuring the frequency response characteristics of the clamped beam while the other pickup (MM 0002) located at $x/l \simeq 0.7$ for measuring the velocity . At a given level of excitation, depending on displacement, velocity and acceleration of the clamped beam it was possible to investigate some of the nonlinear dynamic characteristics of motion. The overall experimental procedure is described in a sequence manner as follows :

4.4.1 Frequency response characteristics of the beam

Based on the theoretical model presented in chapter(3), and for the dimensions and material properties of the test beam. It can be seen that relevant parameters for the test beam are:

$$\bar{P} = PL^4/EIh = 24.17 * a$$

$$I = bh^3/12 = 3.656 * 10^{-15} \ m^4$$

The first three natural frequencies of the beam, f_n ($n = 1-3$) are 190 Hz , 520 Hz and 1020 Hz respectively.

With all these parameters prescribed, the test procedures were carried out as follows :

a) The test beam was firmly fixed to the base frame to simulate the clamped configuration as possible without any initial axial tension or deflection, i.e straight beam .

b) For measurements of the beam response, the contactless pickup (B&K MM 0004) was mounted and fastened by a u-shaped steel plate of 3mm thickness bolted with the base frame . The pickup was located at $x/l \simeq 0.3$ and at distance 1.5mm over the test beam surface to monitor the displacement of the beam at this location. The same procedure was done to the other contactless pickup (B&K MM 0002) from the other end of the clamped portion of the test beam to monitor the velocity of the beam also at $x/l \simeq 0.7$.

c) For measurements of the acceleration of the exciter head , the piezoelectric accelorometer (B&K 4371) was mounted at the center of the base frame which was connected to the vibration meter (B&k2511) previously calibrated to read the peak-peak or the R.M.S value of acceleration in (m/s^2).

d) The base frame carrying test beam , pickups and accelerometer was mounted symmetrically to the exciter head of the vibration exciter (B&K 4808), each pickup was attached to a measuring amplifier (B&K 2610).

e) After the calibration of the instruments, vibration exciter was driven by a power amplifier (B&k2712) and controlled by the exciter control (B&k1047) from which the excitation frequency was changed and the amplitude of excitation was controlled through the output voltage.

f) By varying the excitation frequency from (0-1400)Hz, at a given level of excitation, the vibration signals were averaged on the measuring amplifiers , monitored on the digital frequency analyzer (B&k 2131) and the digital oscilloscope and for certain conditions samples were recorded on the X-Y recorder.

g) During the measurements of the beam response characteristics and chaotic motions, at each given value of the excitation frequency, the level of excitation was increased step-wise within the available instruments range from 5-50 m/s^2 peak to peak. Also tests were reported by keeping force level constant by controlling the amplitude of the output voltage from the vibration exciter and varying the excitation frequency in two directions i.e increasing the frequency and then decreasing it to observe the effect of amplitude on the nonlinear dynamic characteristics such as jump phenomena, sub and super harmonics in addition to the chaotic behavior.

4.4.2 Chaotic motions

To investigate the transition from regular (periodic) to chaotic motions experimentally, observations were made on the conditions under which these motions occur depending on phase plane plots (i.e x versus $\dot{x}$). At certain conditions of frequency and force samples of these were presented graphically.

4.5 REPEATABILITY OF MEASUREMENTS

Measurements of vibration amplitudes were repeated and averaged from time to time under the same test conditions. Their mean was the final recorded value, the average discrepancy was found to be ±6 %.

CHAPTER 5

RESULTS AND DISCUSSION

5.1 OBJECTIVES

The aim of the present experimental work was to investigate the effect of the level of external excitation on the nonlinear dynamic response of the clamped-clamped beam, mainly the sub and super harmonic resonances, jump phenomena. An attempt was made to determine the conditions under which transition from regular or periodic motions to chaotic motions depending on the phase-plane plots. In this chapter, samples of the experimentally obtained results are graphically represented and discussed. Before each experimental run, preliminary tests were made in order to check the satisfaction of the required experimental conditions and the measuring instruments.

5.2 PRELIMINARY TESTS

The first step done in this experimental work, was to ensure that the test beam is firmly fixed to the base frame, taking into account that the test beam is not subjected to an initial deflection or initial axial tension. After this the magnetic transducers and accelorometer were mounted on the specified positions as illustrated in the previous chapter and connected to the vibration measuring instruments. A simple impulsive test response was made to determine the damping coefficient of the beam using the logarithmic decrement equation :

$$\delta = \frac{1}{n} \ln \frac{x_o}{x_n} \qquad (5\text{-}1)$$

where :

x_o = the amplitude of the first single cycle.

x_n = the amplitude of the nth signal cycle.

The free beam oscillations are shown in Fig.(5-1). The logarithmic decrement was found to be about 0.023.

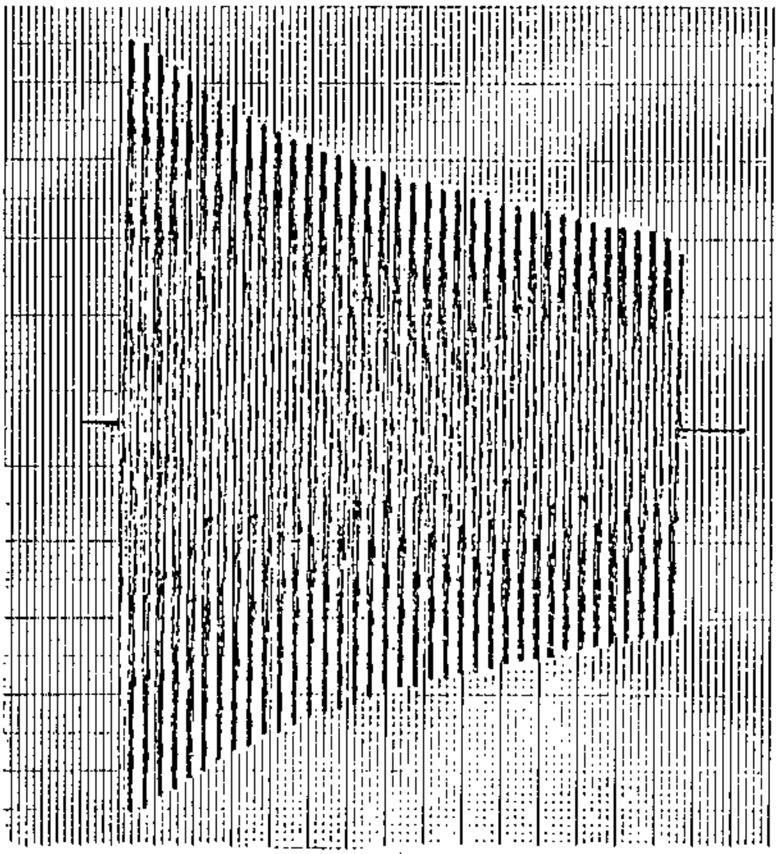


Fig.(5-1): Free oscillations of the test beam after impulse.

5.3 EFFECT OF THE EXCITATION LEVEL ON THE BEAM RESPONSE

The forced beam response was investigated over a wide range of excitation frequencies and through increasing the forcing amplitude step wise at each frequency range. So it was possible after several tests to monitor some of the dynamic characteristics of the beam response at certain frequency ranges and levels of excitation.

5.3.1 Nonlinear frequency response characteristics.

The test was first conducted at frequency range of (0-50)Hz. The dynamic response of the beam was observed to be simple harmonic within the available range of the forcing amplitude. However, in the frequency range (50-65) Hz and at a forcing amplitude $\bar{p}$ below 170 the response was simple harmonic at which the dominant frequency is the external one.Increasing the force level above 170, by increasing the acceleration of the vibration exciter, causes an excitation of other harmonics, mainly the third super harmonic. From 65Hz up to 80Hz, it was noticed that the beam response is simple harmonic at a forcing amplitude $\bar{p}$ below 144. Increasing the amplitude of force above this level,would excite other higher harmonics such as the second and the third super harmonics. Similar behavior was observed at the frequency range from 80Hz up to 95 Hz. However, over this range the response was simple harmonic below the forcing amplitude $\bar{p}$= 70. A jump was observed throughout the frequency range (87 Hz - 93 Hz). Depending on the force level, it was also observed that the third super harmonic resonance of the first mode was the dominant frequency.

The next range of frequencies that was studied is from (100-150) Hz. Throughout this range of frequencies several observations were made :

a) From (100-110)Hz and at $\bar{p}$<144, the response was simple harmonic, at $\bar{p}$=240 the second and third super harmonics of the first mode take place simultaneously, although the external frequency was the predominant.

b) From (110-125) Hz and at $\bar{p}$<100, the response was simple harmonic. Increasing the forcing amplitude step-wise above 100 the second super harmonic of the first mode started to grow while the third super harmonic contribution became insignificant.

c) From (125-150) Hz and at $\bar{p}$<60, the response was simple harmonic. Increasing the amplitude step-wise excited the second super harmonic. A jump was noticed at the frequency range (140-147)Hz at $\bar{p}$= (240 to 450) at which the second super-harmonic resonance frequency was the predominant.

The third range of frequencies studied was near the primary resonance (i.e near the natural frequency of the first mode which is 190 Hz). So, in the range from 155 Hz up to 300

Hz the dynamic response of the beam was harmonic at all force levels. Also no sub or super harmonics were observed, the predominant frequency was the same as the external excitation frequency. A jump was noticed at frequency range from (290-300) Hz preceded by high noise.

The fourth frequency range investigated was the range from 350 Hz up to 600 Hz. Increasing the forcing amplitude above 120 excites the sub harmonic resonance of order 1/2 , and at $\bar{P}>240$ this sub harmonic becomes the dominant one. Ajump was observed at frequencies from 550 Hz to 600 Hz depending on the excitation level .

The fifth frequency range dealt with was from 610 Hz to 800 Hz. A similar behavior to other ranges was observed but at this range a high noise like cracking was clearly heard at $\bar{P}>300$. A jump was also observed through the frequency range from 760 Hz to 800 Hz at which the sub harmonic resonance at order 1/3 was the dominant one .

The final range investigated was from (900-1300) Hz, which is near the third natural frequency of the beam. Increasing the forcing amplitude above 80 step-wise shows that jump would occur at frequencies near 1200Hz at which no dominant sub or super harmonics were noticed.

Samples of results shown graphically in Fig's (5.2-5) for $\bar{P}=60$, 120, 360, and 500 respectively. In these figures, the vertical axis is the rms value of the normalized displacement $y=Y/h$ at $x/l=0.3$ while the horizontal axis is the excitation frequency in Hz. The two numerals in parenthesis indicate the mode vibration and the dominant order of harmonics, respectively.

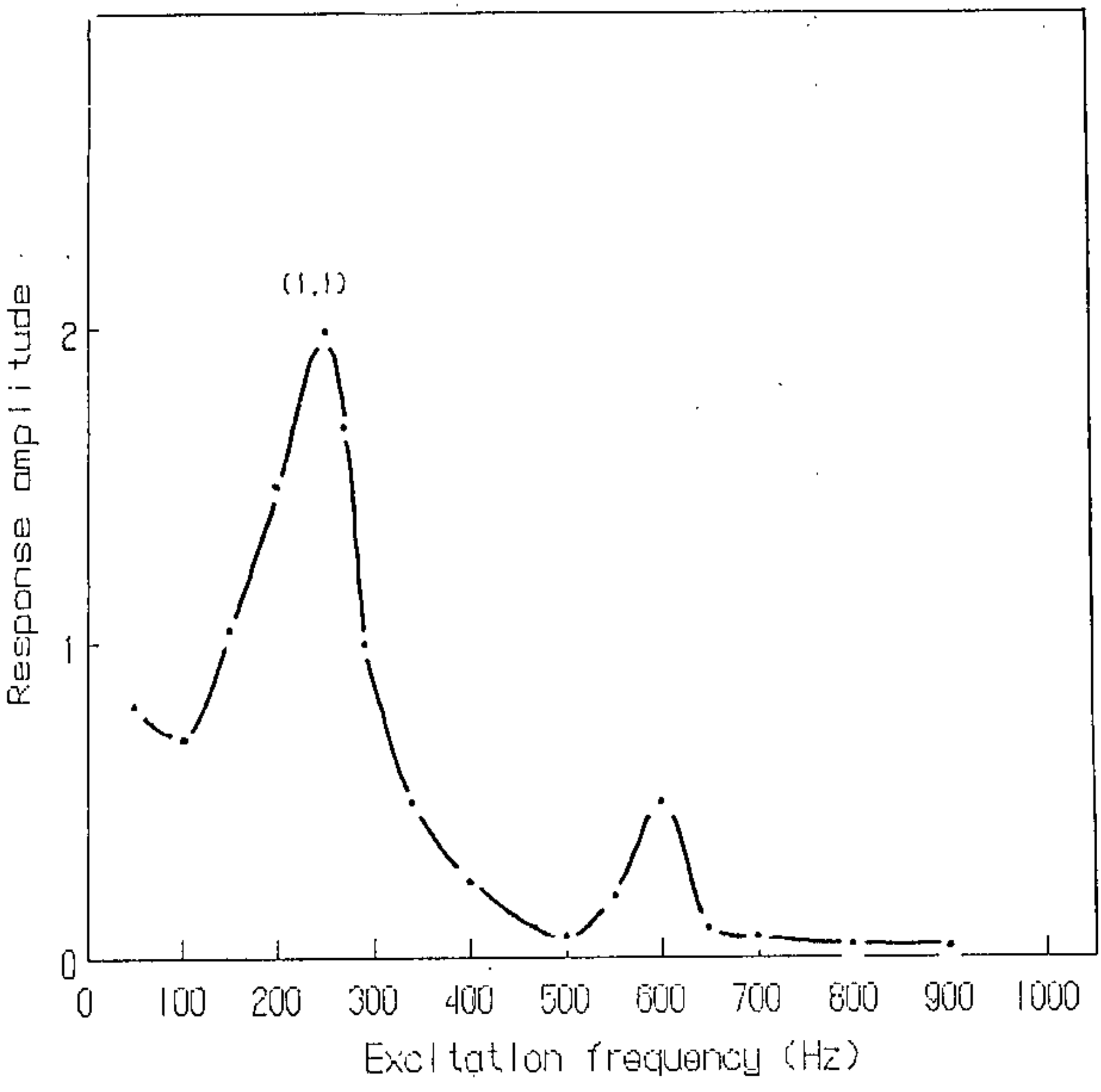


Fig.(5-2): Frequency response characteristics for the test beam at $\bar{p}$ = 60.

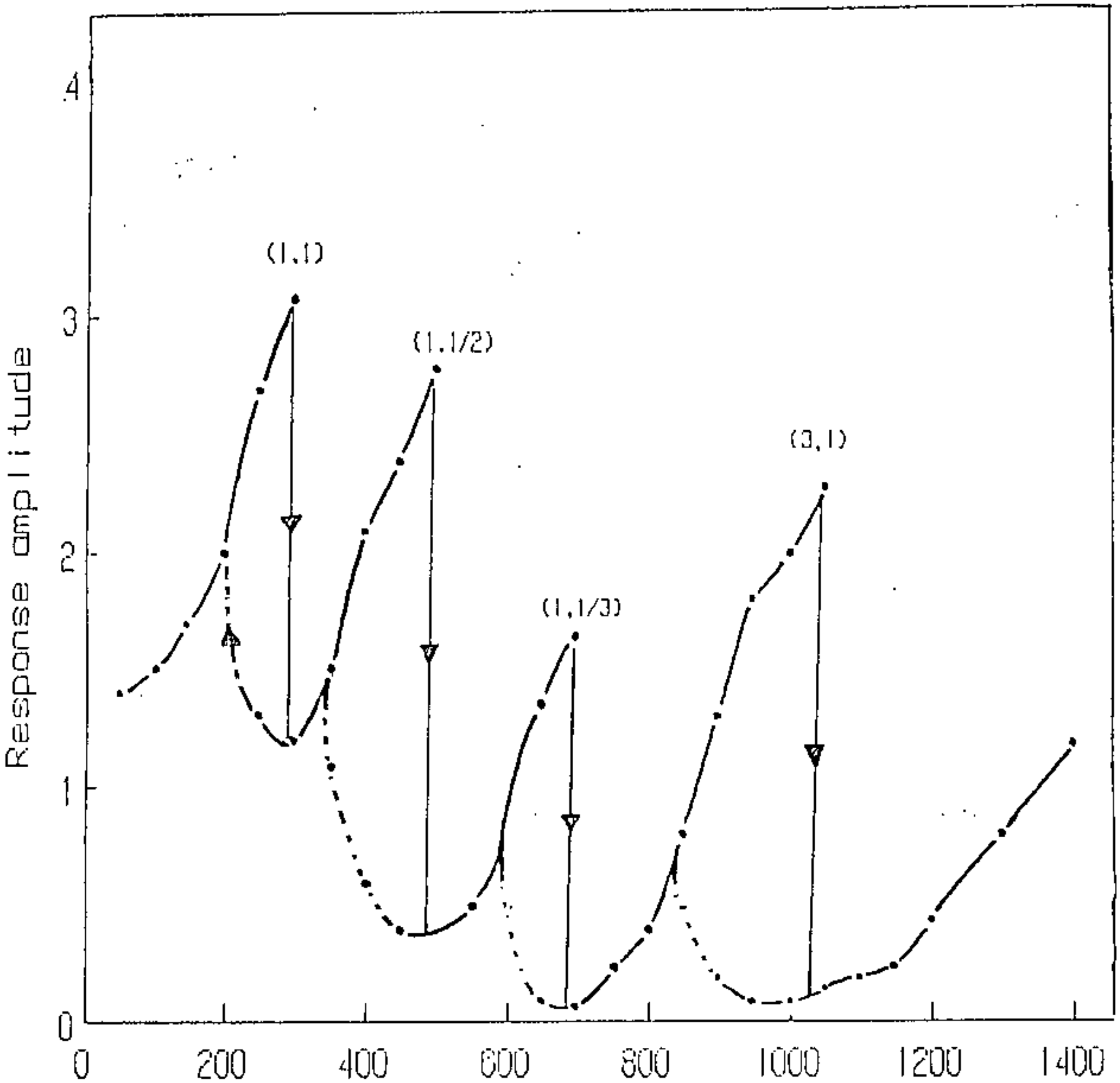


Fig.(5-3): Frequency response characteristics for the test beam at $\bar{p}$ = 120.

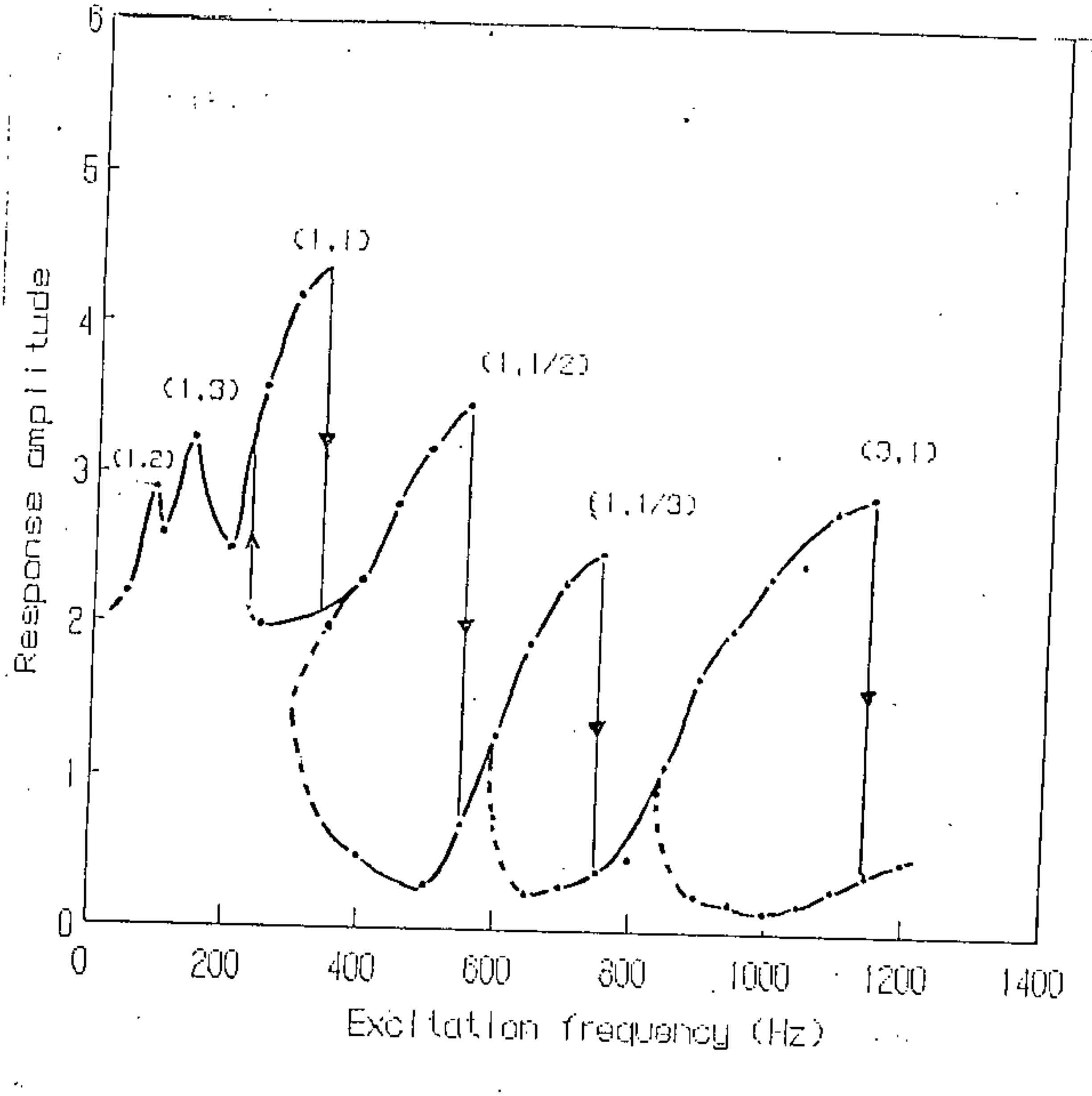


Fig.(5-4): Frequency response characteristics for the test beam at $\bar{p}$ = 360.

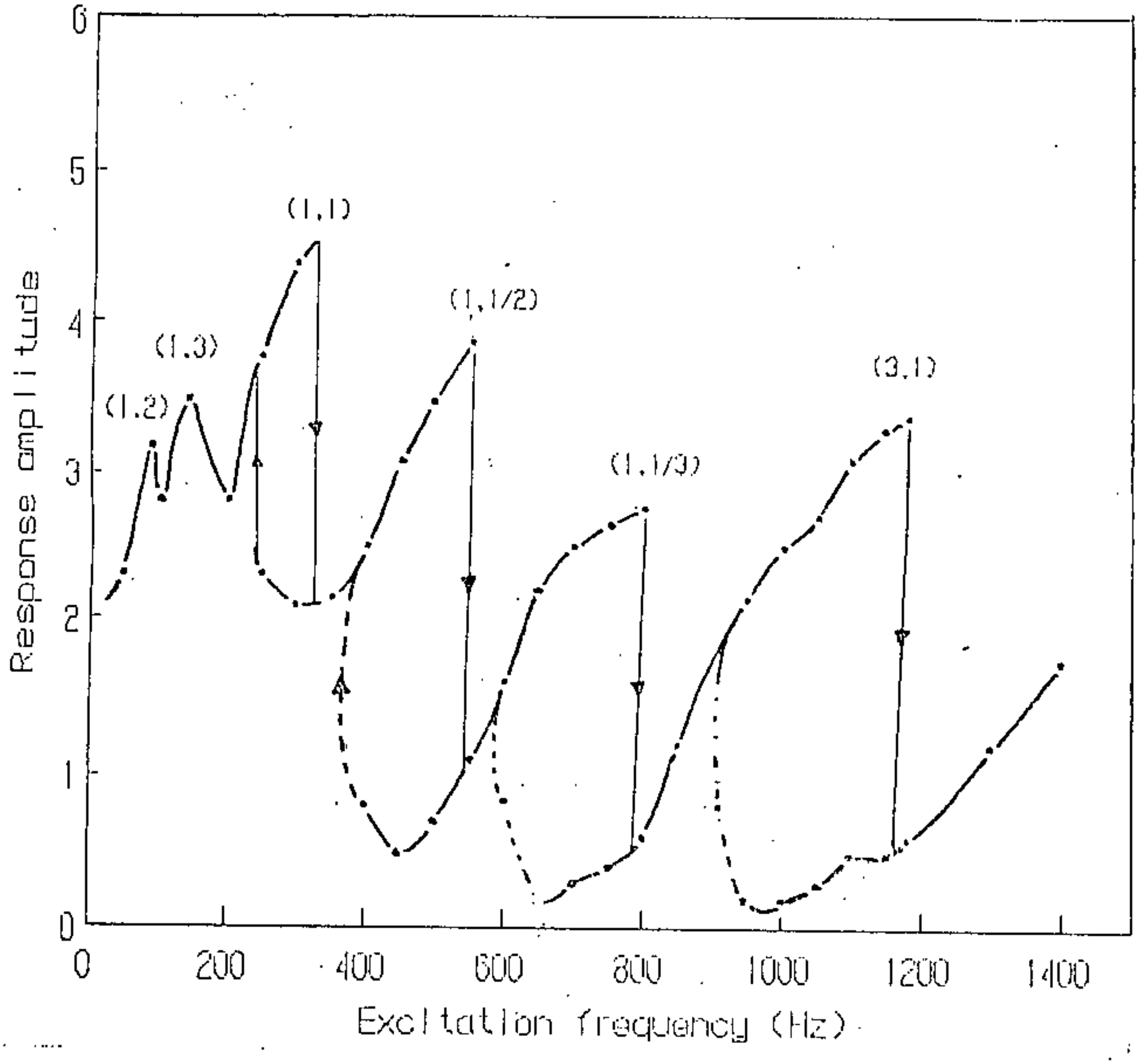


Fig.(5-5): Frequency response characteristics for the test beam at $\ddot{p}$ = 500.

5.3.2 Chaotic motions

In investigating the effect of the large amplitude of excitation on the nonlinear dynamic response of the test beam, an attempt was made to observe the conditions under which the transition from regular to chaotic behavior occurs. So the tests mentioned in the previous section were repeated knowing that from previous studies made on chaotic motions that chaos does usually occur at certain conditions of system parameters such as force level and damping, Ueda (1979), Moon (1980). This transition occurs near the jumps at frequencies which have a vertical tangent on resonance curve, such as the sub-super harmonics, Stupnika and Bajkowski (1986). Another observation was notified, namely chaos is preceded by a cascade of period doubling bifurcation, Fang and Dowell (1987) and broad frequency spectrum . Moon and Li (1990).

As the model investigated here is deterministic in the sense that all parameters in the equation of motion are known ones, the tests were conducted at the frequency ranges (60-95) Hz, (130-150) Hz and in the frequency range (360-570) Hz. These ranges contain jumps at which the dominant frequencies are the sub and super harmonics. Tests were conducted by fixing a certain frequency and changing the level of excitation, the resulting effects are monitored on the phase-plane (displacement vs velocity) plots and time histories of motion. A sample of the results is shown in Fig.(5-6) which was taken at 90Hz. By increasing the amplitude of excitation step-wise, the phase-plane plot undergoes a series of period doubling before chaotic motion occurs. A similar behavior was observed in the second frequency range (130Hz-150Hz) which also leads to n-periodic motions close to chaos. It should be mentioned that these motions as (d) in Fig.(5-6) might not persist, it could suddenly change in the shape, frequency spectra at f=90 Hz and f=145 Hz are shown in figures (5-8) and (5-9) respectively.

Through the frequency range (510-560) Hz, the transition from periodic to chaotic shows a different behavior.First it wasn't preceded by period doubling, and second it occurs at a fixed level of excitation by slightly increasing the forcing frequencies. Increasing the force level, the beam returns to periodic motion again. Samples of these motions are presented in the phase-plane plots and time histories of motion as shown in Fig.(5-7). The frequency spectrum is shown in Fig.(5-10).

Depending on these observations, we tried to propose a criteria for chaos in the frequency domain for different forcing amplitudes as shown in Fig.(5-11). In the regions numbered by the roman numeral III:chaotic motions, were observed to occur, while the regions numbered by the roman numeral II: multi-periodic motion, were noticed,whereas the single periodic motion lies within the regions numbered by I. These conclusions were based on observing the variation of the phase-plane trajectories, time histories of motion in addition to the frequency spectrum.

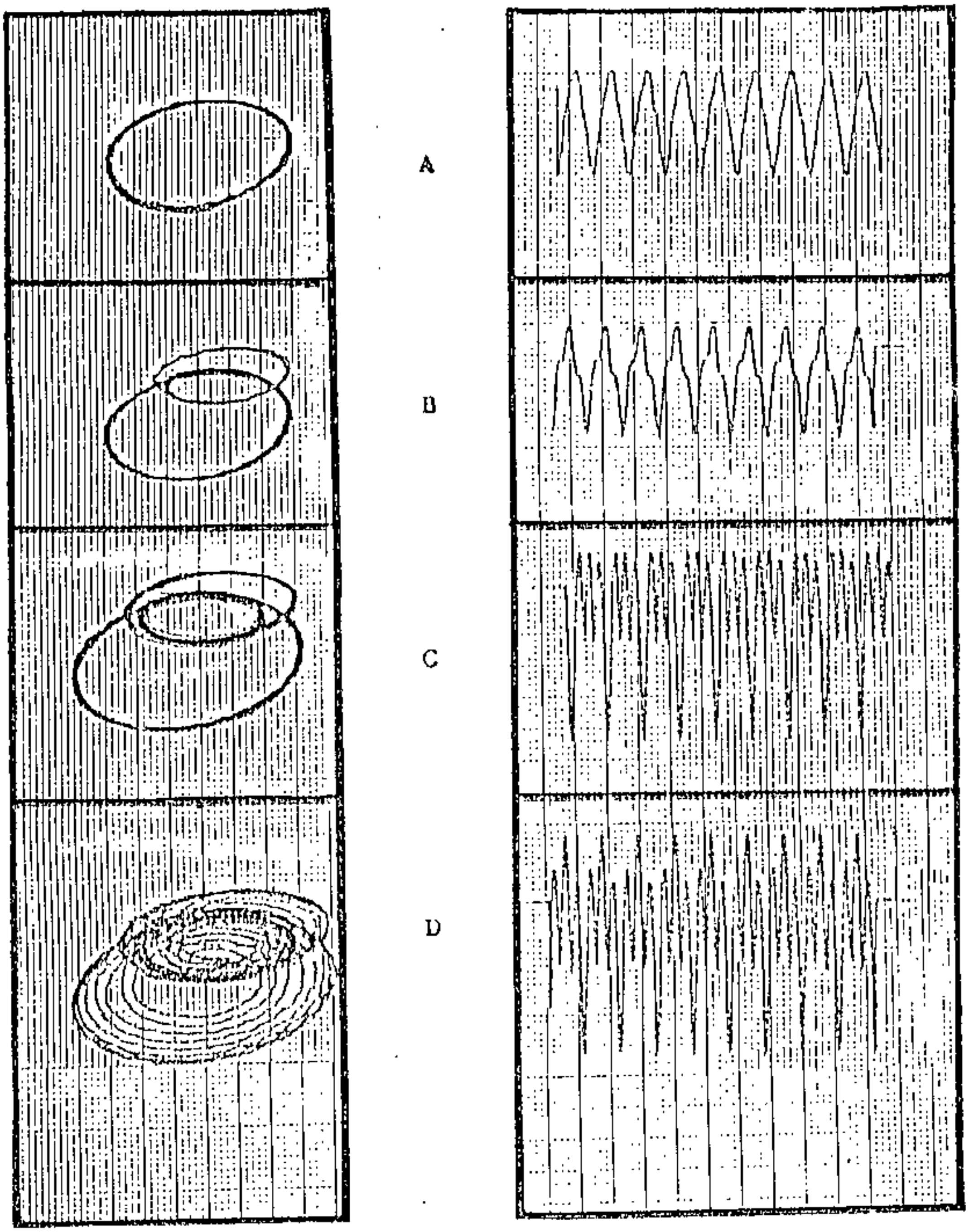


Fig.(5-6): Phase-plane plots and time histories of motion at 90 Hz and at $\bar{p}$ = a) 50 b)120 c)360 d)500 respectively.

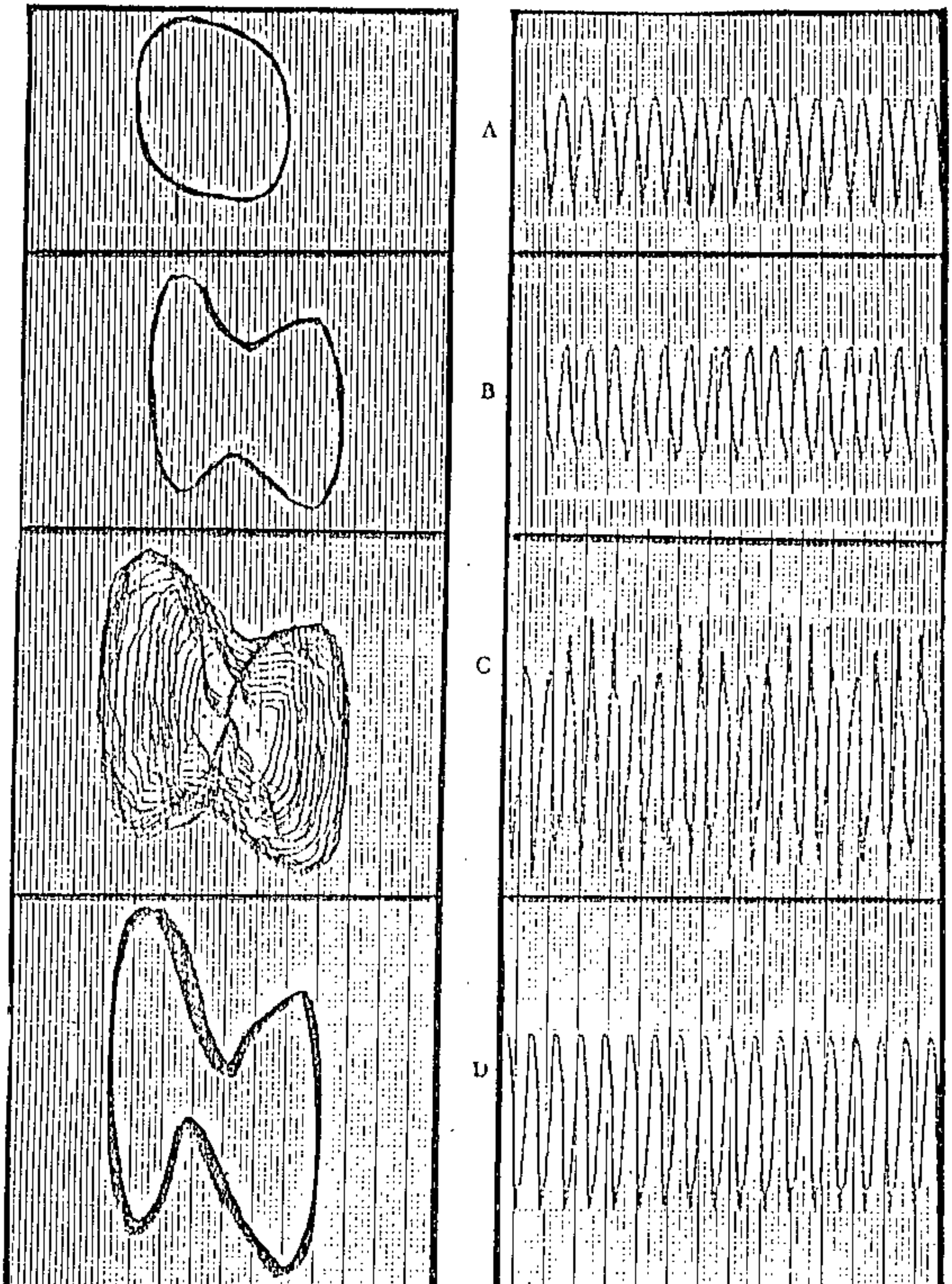


Fig.(5-7): Phase-plane plots and time histories of motion at 540 Hz and at $\bar{p}$ = a) 50 b) 80 c)110 d) 140 respectively.

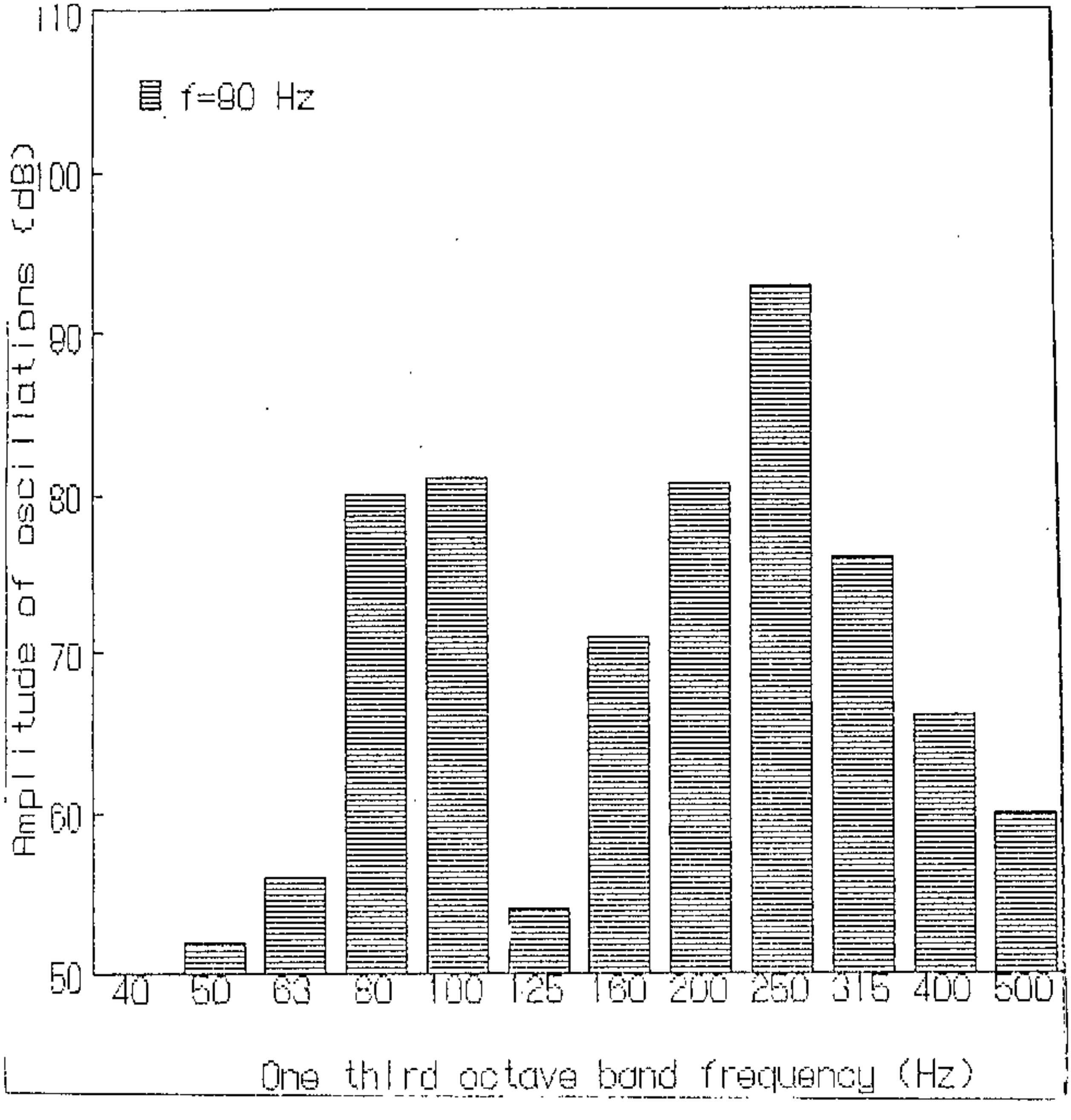


Fig.(5-8): Frequency spectrum of oscillation signal at 90 Hz and $\bar{p}$ = 500.

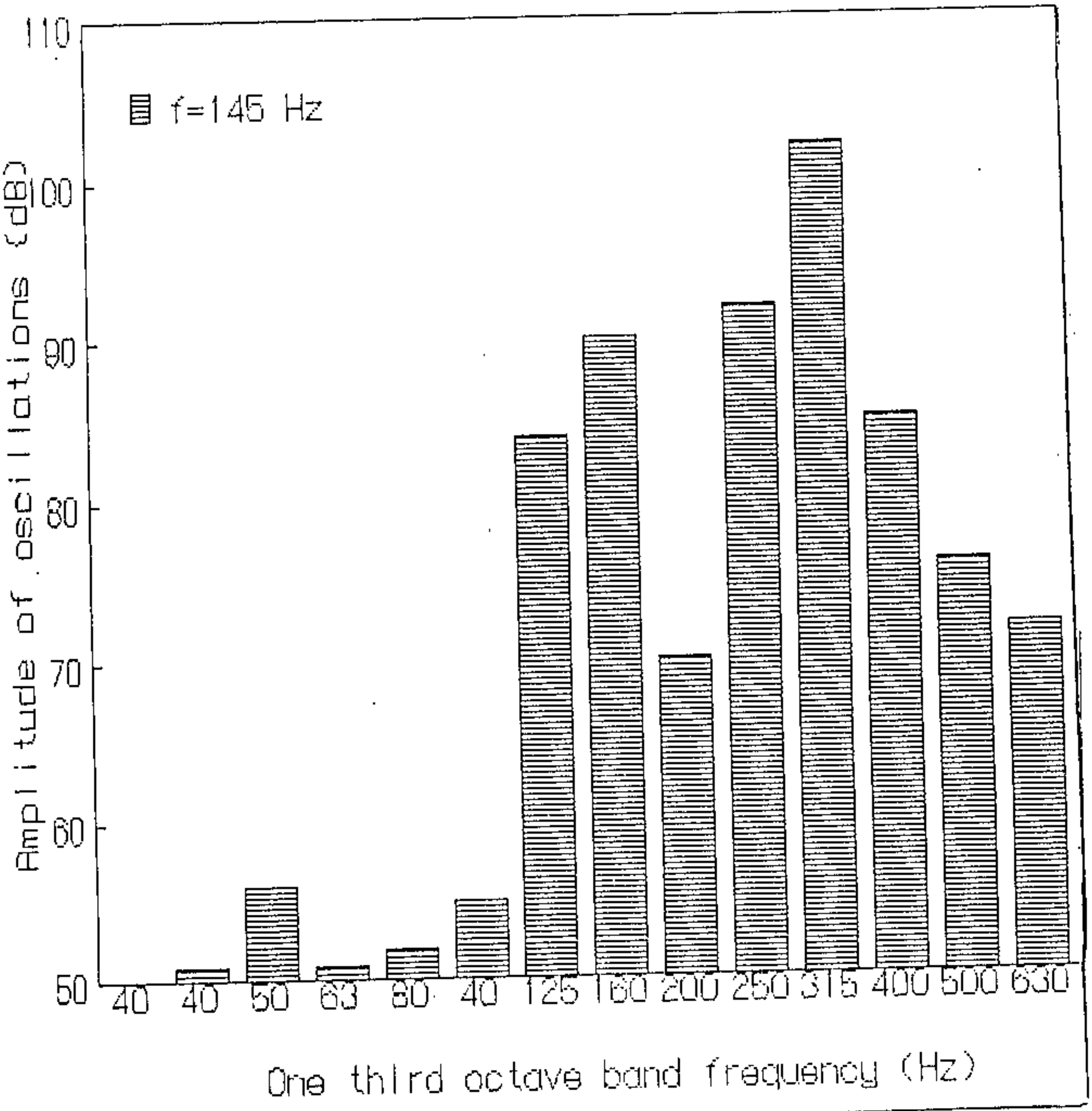


Fig.(5-9): Frequency spectrum of oscillation signal at 145 Hz and $\bar{p}$ = 500.

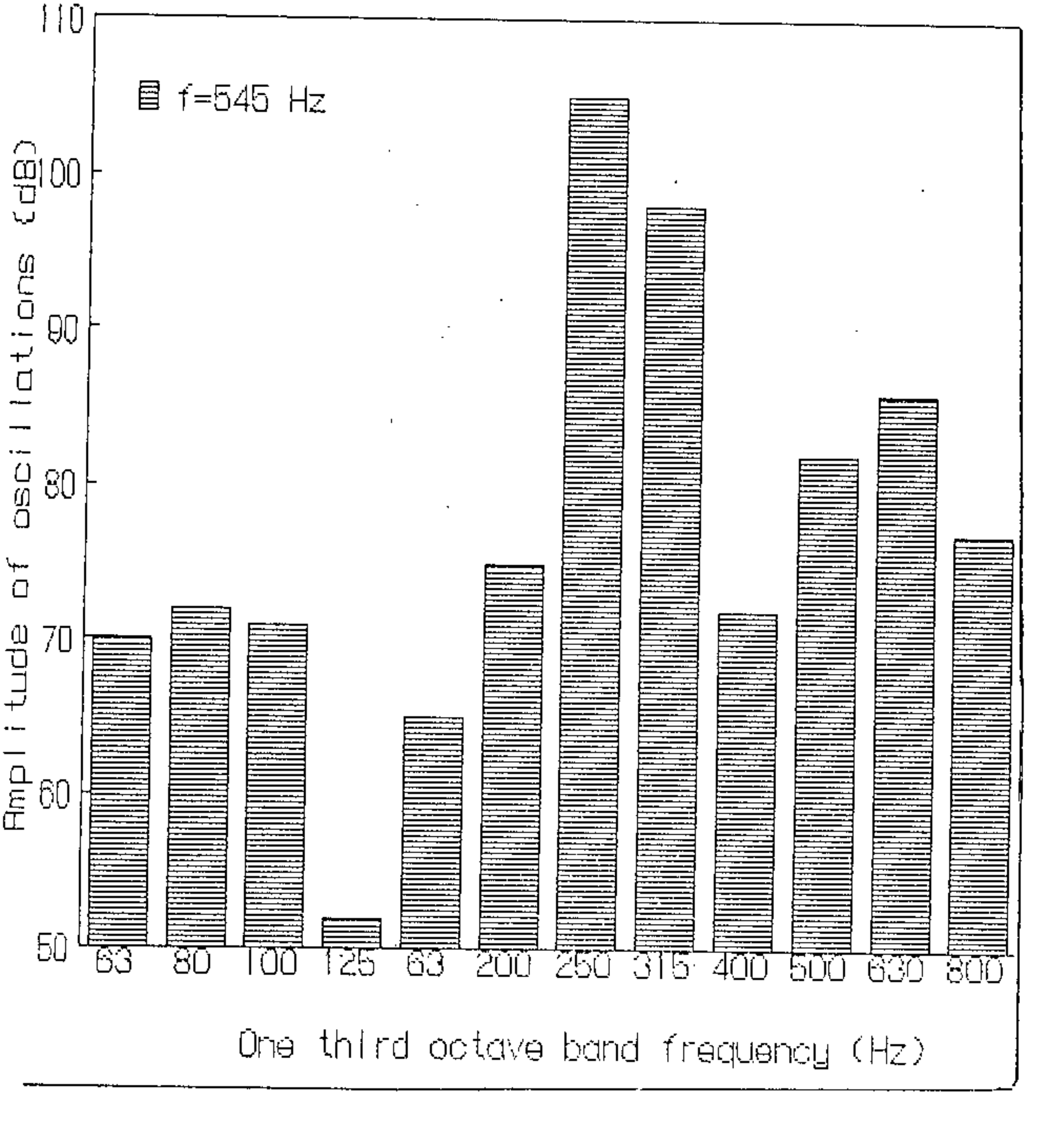


Fig.(5-10): Frequency spectrum of oscillation signal at 540 Hz and $\bar{p}$ = 100.

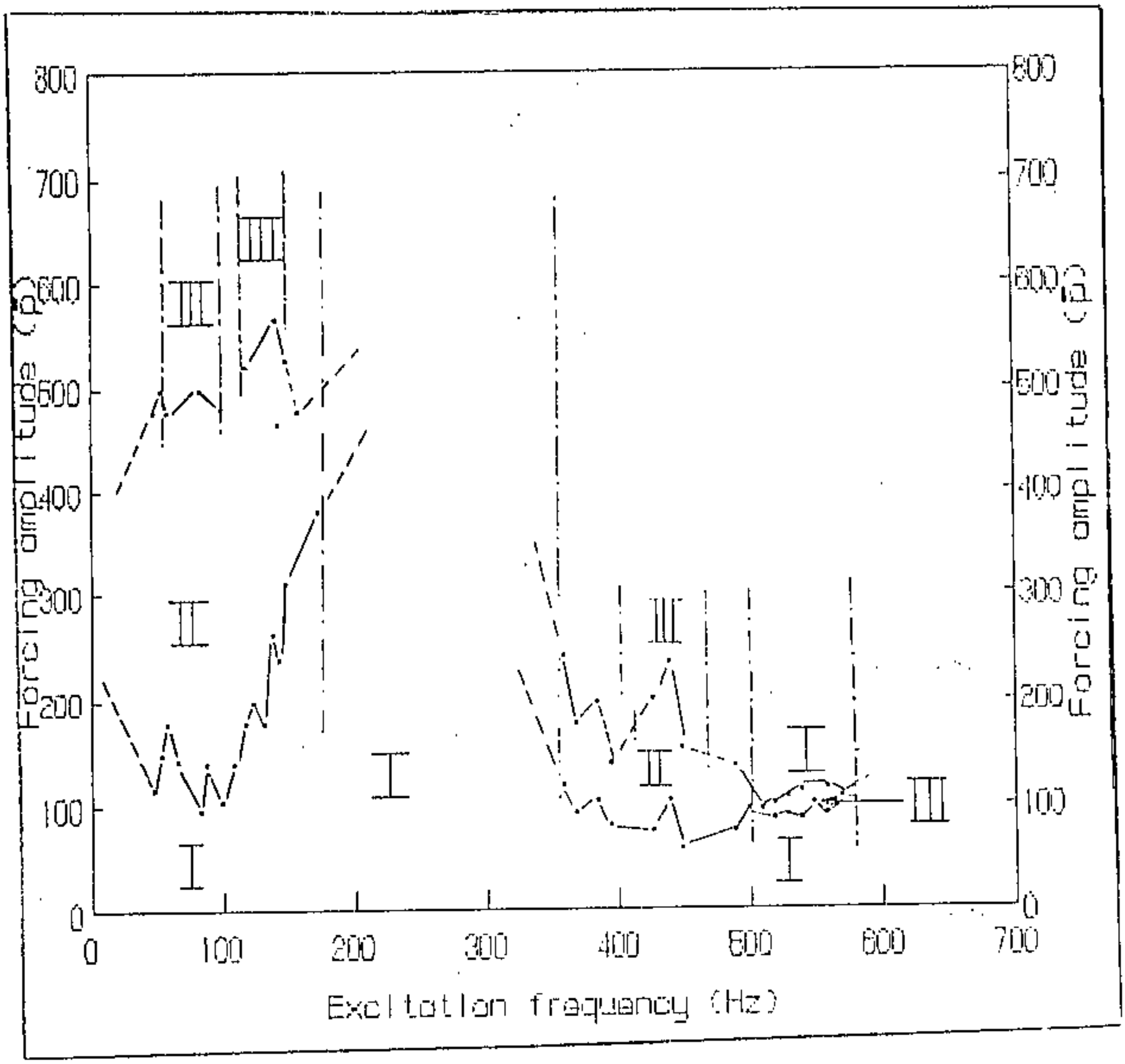


Fig.(5-11): Experimental criteria for chaos in the frequency domain for different forcing amplitudes of the test beam .

Throughout the region in the neighborhood of the primary resonance region no chaotic motion was recorded and the response can be considered, to some extent, to be simple harmonic at the available levels of excitation. Another behavior was noticed at certain frequency ranges in particular at 650 Hz , that motion may change from single periodic to multi-periodic then to chaotic without changing the level of excitation or the excitation frequency. When the system is left to vibrate, then after some time, these changes in motion occur suddenly. Sample of this behavior is shown in Fig.(5-12) in which this transition from single to multi-periodic occurs after 20 minutes.

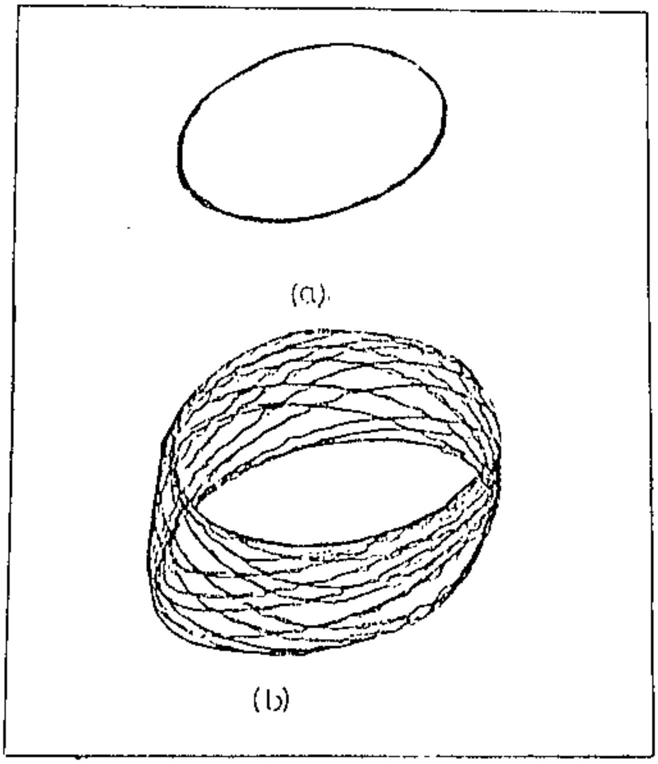


Fig.(5-12): Transition from periodic in (a) to multi-periodic in (b) after 20 minutes at f=650 Hz, $\bar{p}$=120.

Finally, the variation of the second and third harmonic amplitudes were plotted in the frequency response curves at $\bar{P}$ = 500 as shown in Fig.(5-13).

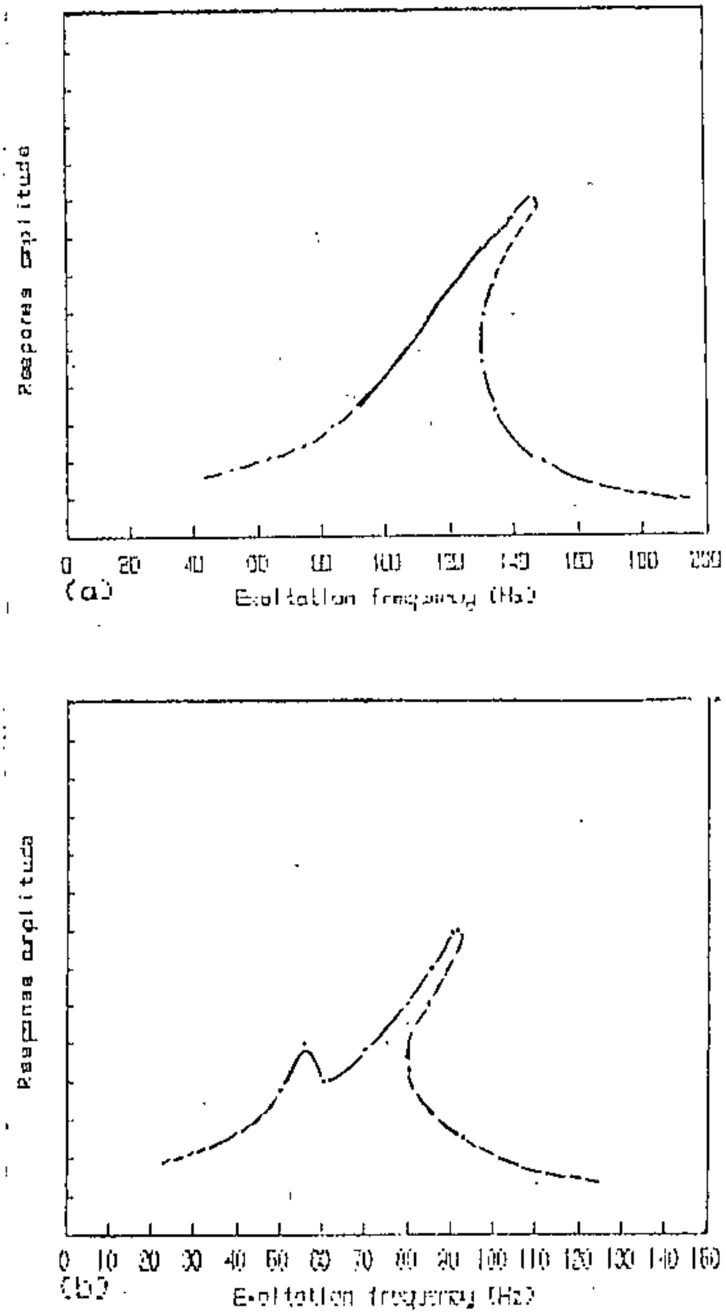


Fig.(5-13): Frequency response curves at $\bar{p}$ = 500 for (a) second harmonic (b) third harmonic amplitude respectively.

Discussion

During the previous tests which were mainly conducted to investigate the effects of level of excitation on the nonlinear beam dynamic response, it can be inferred that the forcing amplitude $\bar{p}$ plays a significant role in the determination of many response characteristics, especially chaos and sub-super harmonics. Although the forcing amplitude was limited, the observations and results of the beam frequency response characteristics were found to be in some agreement with the experimental and theoretical work of Takahashi (1979) except the fifth harmonic of the third mode and the third harmonic of the second mode were not observed in this work. Also the present study somehow shows a similar trend to that reported by Yamaki and Mori (1980), but combination resonance of the first and third modes at the first harmonic was not observed within the available range of forcing amplitude. Regarding the observation on chaotic response within the specified frequency domains and forcing amplitudes, taking into account the limited range of external excitation, the transition from regular to chaotic motions in the regions below the first linear resonant frequency is preceded by a series of period doubling observed in the phase-plane, accompanied by a distorted wave forms of time histories of motion and broad banded frequency spectrum, similar to some extent to those reported by Moon Shaw (1982). In the frequency ranges above the primary resonance, chaotic response was recorded and appeared not to be preceded by a cascade of doubling periods, but appeared only and persist at a certain forcing level. Increasing or decreasing the level of excitation above or below this range this motion ceases to appear. As was mentioned previously, this range supports the 1/2 sub harmonic resonant frequency, which is in agreement to the range reported by Stupnicka (1987) found in the analog and digital computer results.

CHAPTER 6

CONCLUSIONS

The objectives of the present work, set in chapter 1, were to investigate experimentally the effect of the excitation level on some of the nonlinear dynamic characteristics of a clamped beam. This chapter is devoted to summarize the work done on the thesis, to outline the points which have emerged from the present investigation and to recommend some points on this problem for further investigation.

6.1 SUMMARY OF THE WORK

The review of previous studies on the nonlinear response of a system modeled by Duffing's equation, revealed the absence of experimental work on some aspects like the chaotic response. Most of these investigations were dependent on analog and digital computer results which appear from the numerical solutions of Duffing's equation. The present investigation was directed toward exploring the effect of the forcing amplitude on some nonlinear characteristics of a practical model found in many real life situations, a beam element with a clamped-clamped configuration.

6.2 CONCLUSIONS

Several points have emerged from this experimental work on the effect of the excitation level on the nonlinear dynamic response of clamped beam. These can be summarized as follows :

a) The beam element exhibited almost a linear response at all frequency ranges characterized by simple harmonic motions and single periodic solutions, when the forcing amplitude was less than 50. While at certain regions of frequency which is in the neighborhood of the fundamental beam frequency the response was observed to be purely harmonic at all available forcing levels.

b) Increasing the forcing amplitude, of course, by controlling the acceleration of base frame and hence the test beam, the nonlinear behavior was characterized by many amplitudes jumps at different excitation frequencies in the increasing and decreasing directions. Sub-super harmonics contribution leading to multi-periodic motions, the third and second super harmonics of the first mode were observed below the primary resonance, which shows a similar trend reported by Takahashi (1979), Yamaki (1980), Roberts (1984). The 1/2 and 1/3 sub harmonic response appeared above the primary response i.e, at frequencies above the natural frequency of the first mode.

c) The transition from regular to chaotic motions occursnear the amplitude jump of the 1/2 sub harmonics response within a certain range of the forcing amplitude, which is in agreement to the results of Ueda (1979), Stupnicka (1987). In the third and second super harmonic resonance a series of periodic doubling occurs at increasing force levels in the route to chaos.

d) Differences among investigators results, may be referred to the differences in the method of study, whether it was a computer simulation or experimental investigation and also in the experimental conditions, such as, method of beam support, damping, material and methods of measurements.

e) There is a close connection between the forcing level amplitude jumps sub and super harmonics to the occurrence of chaotic response of this nonlinear model.

6.4 RECOMMENDATIONS FOR FUTURE WORK

The review of previous work presented in chapter 2 with the present experimental work, suggest that more experimental work is needed to understand the different aspects arise in the nonlinear forced vibration. The area of research, that might be needed, for further experimental investigations :

a) Experiments on a model with multi-degree of freedom or with n coupled modes of vibration, since chaotic response has not been thoroughly studied in systems with more than one degree of freedom.

b) A test beam model with different cross sectional areas with different types of excitations, i.e unsymmetric and symmetric, and different boundary conditions, have to be investigated in a combination of these conditions.

c) Experiments on a real structures in space such as the practical truss-type construction in which the looseness of joints may bring in nonlinearities which have to be investigated using new tools instead of the linear methods to understand the dynamics of such structures.

d) The contribution of other nonlinear vibration modes such as torsional, axial in addition to the flexural forced vibration simultaneously for the test beam, has to be thoroughly performed on a proper model.

e) A threshold criteria for chaotic response that can be valid for different cases for nonlinear vibrating systems, have not been fully developed, the exciting ones merely valid for specified cases.

f) A study of the effect of damping on chaos and whether it may be possible to construct certain devices to avoid occurance of chaos or at least to diminish their negative effects.

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ملخص الرسالة

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مميزات الاستجابة الديناميكية غير الخطية لعارضة مثبتة معرضه لمؤثر دورى موزع بشكل منتظم .

تعتبر دراسة الاجزاء الهندسية المعرضة للاهتزازات ذات أهمية كبيرة من الناحية العلمية ، وذلك لتعرضها لاهتزازات اما نتيجة لظروف العمل المحيطة أو كنتيجة لتأثيرات خارجية مثل قضبان وعوارض الانشاءات المعرضة لاهتزازات طولية أو انحنائية أو كأعمدة نقل الحركة المعرضه لاهتزازات طولية التوائية أثناء دورانها .

وحيث أن العوارض تستخدم في الكثير من التراكيب الهندسية ، لذلك فأن دراسة السلوك الديناميكي لهذه التراكيب الهندسية للمؤثرات الخارجية ذات أهمية مميزة خاصة عندما يبدأ السلوك في التحول من السلوك الخطي الى السلوك غير الخطي .

تكمن دراسة الظاهرة غير الخطية في العوارض المهتزة في وجود العديد من الظواهر التي لا تلاحظ عند دراسة السلوك الخطي مثل ظاهرة القفزات وظهور كسور ومضاعفات للذبذبات المحدثة للاهتزازات ، وكذلك الظاهرة العشوائية ولذلك فأن الاعتماد على النظرية غير الخطية لدراسة وتحليل هذه الظواهر يصبح ضروريا لتنبؤات أكثر واقعية في معرفة كيفية الاداء الديناميكي لهذه العوارض ومدى استمراريتها دون حصول فشل في الاداء قبل الاوان بشكل غير متوقع .

في هذا البحث تمت دراسة معملية للاستجابة غير الخطية لعارضة مثبته من كلا جانبيها معرضةلمؤثر خارجي دوري منتظم ، بشكل خاص تمت دراسة تأثير زيادة التأثير الخارجي على حدوث الظواهر غير الخطية للعارضة المهتزة وكذلك تم اشتقاق المعادلات النظرية التقريبية الممثلة لسلوك هذه العارضة المشابهه لنموذج (دفنج) وقد أجريت التجارب في هذا البحث على عارضة من الفولاذ الزنبركي ١٠٥ مم × ١٣ مم × ١٥ر٠ مم وذلك على مدى واسع من الذبذبات يتراوح من (صفر - ١٤٠٠) هيرتز على مستويات مختلفة من المؤثرات الخارجية ، وقد قيست استجابة العارضة بقياس

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الازاحة النسبية للعارضة بالنسبة لقاعدة العارضة .

ومن أهم النتائج التي توصل اليها البحث ما يلي :-

١. ان سلوك العارضة يكون خطيا عند مستوى التأثير دون (٥٠) ، وكذلك بالقرب من الرنين الاساسي للعارضة أي بالمجال المجاور للذبذبة الاولى الطبيعية للعارضة المهتزة وكانت حوالي (١٩٠) هيرتز .

٢. ان زيادة مستوى التأثير يؤدى الى تحول سلوك العارضة للسلوك غير الخطي الذى يلاحظ بحدوث القفزات في ازاحة العارضة وظهور مضاعفات للذبذبة المحدثة للاهتزازات في المجال الذى يسبق الرنين الاساسي للعارضة وخصوصا ضعفي وثلاثة أضعاف الذبذبة ، بينما ظهرت أجزاء من التوافقية في المجال الذى يأتي بعد الرنين الاساسي للعارضة وقد جاءت بعض هذه الملاحظات موافقة لبعض نتائج الباحثين .

٣. الانتقال من الحركة المنتظمة للحركة العشوائية يحدث على منحنى الذبذبة - الاستجابة في المناطق التي كانت تظهر بها نصف النسبة وثلاثة أضعاف وضعفي النسبة للذبذبة المحدثة للاهتزاز وعلى مستوى معين من التأثير الخارجي .

٤. توجد علاقة وثيقة بين مستوى التأثير الخارجي ومناطق القفزات واجزاء ومضاعفات الذبذبات وبين حدوث الاستجابة العشوائية للنموذج المستخدم في هذا البحث .